

Could quantum mechanics, and even gravity, be all about a correct resolution of the classical self-force problem?

Yehonatan Knoll

Email: yonatan2806@gmail.com

Abstract

The self-force problem of classical electrodynamics has two closely linked facets: The ill defined dynamics of a point charge due to the divergent self field at the position of the charge, and the divergence of formally conserved quantities, such as the energy, associated with symmetries of the corresponding Lagrangian. Fixing the self-force problem amounts to the construction of a *new* theory, which is free of the above pathologies and yet “sufficiently close” to the immensely successful original. In a recent paper by the present author [5], such a proposal, dubbed extended charge dynamics (ECD), was presented. The essential ingredients of classical electrodynamics preserved by ECD (and, among the plethora of solutions to the problem, only by ECD) are:

Ontology. The electromagnetic field is the same unquantized classical field, while charges are sufficiently localized conserved currents, accounting for the manifest corpuscular nature of elementary charges.

Symmetries. ECD enjoys the full symmetry group of classical electrodynamics, most importantly the hidden symmetry of scale covariance.

Conservation laws. All ECD conservation laws formally coincide with their classical counterparts, and yet lead to finite conserved quantities.

Despite this seemingly classical setting, and the reduction of ECD to classical electrodynamics in the latter’s domain of validity, it is shown in the present paper that ensembles of ECD solutions could, in principle, reproduce the statistical predictions of quantum mechanics. Exclusively quantum mechanical concepts, such as interference, violations of Bell’s inequalities, spin and even photons (despite the use of a classical EM field), all emerge as mere statistical manifestations of the self interaction of ECD charges. Moreover, ECD, being a single-system-theory, is not merely an interpretation of relativistic quantum mechanics, but rather holds an independent testable content, possibly unifying all forms of interaction including gravity. Much of the imbroglio and stagnation of modern physics may be due to a ‘wrong turn’ taken over a century ago with regard to the self force problem, and exponentiated ever since.

1 Introduction

At the turn of the 20th century there was an absolute ruler to theoretical physics — classical electrodynamics. It was remarkably compact in its formulation, rather accurate in its domain of application, and even compatible with the newly discovered theory of relativity. There remained, however, two problems with classical electrodynamics: It was not even a theory and, when applied to small scales, it was not even wrong (to paraphrase Pauli). It was not even a theory because of the self-force problem — the ill defined Lorentz self-force at the position of the charge — and it was not even wrong because, even when self interaction

is ignored, or rather *because* it is ignored (as ignoring the self interaction is just one possible solution to the self-force problem) there is no model of matter based on classical electrodynamics with which to confront experimental results, such as the strength of chemical bonds or the spectrum emitted by a heated gas.

The apparent connection between the two problems notwithstanding, two distinct paths were pursued in solving each. Quantum mechanics gave concrete and increasingly accurate predictions in circumstances in which classical electrodynamics was silent, but at the same time demolished the clear, and thoroughly tested ontology of the latter — the manifest corpuscular character of elementary charges and the continuous nature of the electromagnetic field. The resolution of the self-force problem followed a completely different, largely pedagogical, rout. Many extensions and deformations of classical electrodynamics were proposed (and still are, with focus shifting to the gravitational self-force problem) all retaining the classical ontology of point-like charges and a continuous EM fields. All modified theories, however, result in some local perturbation to the Lorentz force, and are therefore incapable of explaining the nonlocal aspects of QM.

A century after the above bifurcation took place, equipped with the vast mathematical arsenal accumulated thereafter, we return in this paper to those early 20th century days. Guided by the principle of scale covariance — a hidden symmetry of classical electrodynamics on equal footing with Poincaré covariance — it is shown that the two fundamental problems of classical electrodynamics should, and apparently can, be solved at once. This conclusion stems from a deeper analysis of a recent work by the current author (with Yavneh; [5]), proposing a Lorentz and scale covariant deformation of classical electrodynamics, depending on a small dimensionless quantum parameter. In that deformation, dubbed extended charge dynamics (ECD), particles enjoy a dual local nonlocal character. The local aspect explains, for example, the thin traces left by charges in bubble chambers, and yet leads to a well defined self interaction. The nonlocal part — an inevitable consequence of Lorentz covariance — accounts for quantum mechanical nonlocality which, for example, enables an ECD particle passing through one slit in a double-slit apparatus, to ‘remotely sense’ the status of the other one. This feeble remote sensing mechanism, it is speculated, is also behind gravitational interaction, amplified this time by a huge mass rather than by the huge distance between the double-slit and the detection screen. From yet a wider perspective, gravitation may be just another facet of entanglement.

The possibility of unifying electrodynamics with such a seemingly unrelated theory as gravity, extends also to small scales. The self force in classical electrodynamics is generally ignored as a first approximation, not because it is small (in fact, in the vicinity of the world line of a point charge, the EM field becomes arbitrarily strong) but rather because it leads to fairly accurate results when applied to charges in a slowly varying EM field. The colossal failure of this approximation at small scales, rather than implying the breakdown of the approximation, was taken as an indication for the breakdown of classical electrodynamics altogether at small scales, inviting other modes of interaction, such as the strong force, into the arena of small scale physics. It therefore seems only logical to first have a fully consistent classical electrodynamics, free of self-interaction problems, and only then confront

it with small scale observations. One possibility, for example, suggested by ECD is that two sufficiently close, positively charged ECD particles, need not repel each other, rendering the strong force just a small scale feature of ECD.

It is argued that quantum mechanics (QM), including quantum field theory, only describes certain statistical aspects of ensembles of ECD solutions. And indeed, QM, with its built-in unitarity, deals mainly with statistical questions (S-matrices, reaction cross sections, thermal properties of matter etc.). Over the years, one must acknowledge, some more ‘deterministic’ applications of QM have emerged — determining the strength of a chemical bond, or the mass of a subatomic particle — but it seems that these deterministic uses were ‘forced’ on a statistical theory, in the absence of any alternative method. In those applications, QM does not really provide a satisfactory ontology for the microworld, but rather a set of heuristics surviving experimental tests. ECD, on the other hand, just like classical electrodynamics, is a single-system-theory, dealing explicitly with such deterministic question and, as will transpire, holds the potential for a wide range of predictions. ECD, then, is *not* another interpretation of QM, but rather an independent theory, supposedly compatible with the statistical predictions of QM.

Section 3 of this paper deals with this compatibility conjecture. The classical ontology of ECD notwithstanding, it is demonstrated in representative cases, how various features of ECD render possible the realization of the quantum mechanical predications by means of an ensemble of ECD solutions. The exact nature of the ensemble relevant to the experiment, it is argued, is an independent law of nature, on equal footing with ECD itself. QM therefore describes some statistical attributes of those ensembles, and like the nature of the ensemble itself, enjoys the status of an independent law, complementing ECD rather than rivaling it.

In section 2 (and in the appendix), the mathematical structure of ECD is analyzed in greater depth. The ECD equations derived in [5], originate from a brutally formal Lagrangian, involving ‘delta-function potentials moving in Minkowski’s space’. Such formal objects never come equipped with a precise meaning which is to be determined only by the global consistency of the mathematical structure resulting from their definition. Indeed, such global considerations lead to a ‘refined’ definition of the ECD equations, compared with their form in [5]. One consequence of this refinement, along with another central theme of ECD — scale covariance — is the ability of ECD solutions to ‘drift in scale’. Such a scale drift should be extremely slow on our native time-scale, but could manifest itself on cosmological scales. This speculation, as well as many other complementary remarks, can be found in the many footnotes appearing in this paper, and may be ignored on first reading.

Although acquaintance with [5] may provide some intuition into the origin of ECD, the present paper is entirely self contained, requiring mainly standard graduate level background for its understanding.

2 Extended Charge Dynamics

A note about dimensions. Throughout this paper, functions defined on Minkowski’s space-time, \mathbb{M} , have their values in the relevant abstract mathematical space, viz. no ‘di-

mension' (in the usual sense of mass, length, mass/length etc.) is attached to those objects. In particular, points in space-time are indexed by four labels — real numbers. For the sake of eliminating many constants from all the equations, the labeling convention is chosen so that the speed of light equals 1 everywhere, and in all directions. This determines the labeling of space-time up to an arbitrary scale factor. Scale covariance, established in the sequel, guarantees that this scale factor can remain arbitrary without affecting any *observation*, which is always a pure (real) number, expressing the ratio between two quantities from the same category (this category is traditionally indexed by its 'dimension').

2.1 Manifestly scale covariant classical electrodynamics

There are two components in classical electrodynamics of n interacting charges. One is the Lorentz force, governing the motion of a charge in a fixed EM field

$$\ddot{\gamma}^\mu = q F^\mu{}_\nu \dot{\gamma}^\nu, \quad (1)$$

with $\gamma(s) \equiv \gamma_s : \mathbb{R} \mapsto \mathbb{M}$ the world line of a charge, parametrized by the Lorentz scalar s , q a coupling constant and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the antisymmetric Faraday tensor. Multiplying both sides by $\dot{\gamma}_\mu$ and using the antisymmetry of F , we get that $\frac{d}{ds}\dot{\gamma}^2 = 0$, hence $\dot{\gamma}^2$ is conserved by the s -evolution. This is a direct consequence of the s -independence of the Lorentz force, and can also be expressed as the conservation of a 'mass-squared current'

$$b(x) = \int_{-\infty}^{\infty} ds \delta^{(4)}(x - \gamma_s) \dot{\gamma}_s^2 \dot{\gamma}_s. \quad (2)$$

Defining $m = (\int d^3\mathbf{x} b^0)^{1/2} \equiv \sqrt{\dot{\gamma}^2} \equiv \frac{d\tau}{ds}$ with $\tau = \int^s \sqrt{(d\gamma)^2}$ the proper-time, equation (1) takes the familiar form

$$m\ddot{x}^\mu = q F^\mu{}_\nu \dot{x}^\nu, \quad (3)$$

with $x(\tau) = \gamma(s(\tau))$ above standing for the same world-line parametrized by proper-time. We see that the (conserved) effective mass m emerges as a constant of motion associated with a particular solution rather than entering the equations as a fixed parameter. Equation (1), however, is more general than (3), and supports solutions conserving a negative $\dot{\gamma}^2$ (tachyons — irrespective of their questionable reality).

The second ingredient of classical electrodynamics is Maxwell's inhomogeneous equations, prescribing an EM potential given the world-lines of all charges

$$\partial_\nu F^{\nu\mu} \equiv \square A^\mu - \partial^\mu(\partial \cdot A) = \sum_{k=1}^n k j^\mu, \quad (4)$$

with

$$^k j(x) = q \int_{-\infty}^{\infty} ds \delta^{(4)}(x - {}^k \gamma_s) {}^k \dot{\gamma}_s \quad (5)$$

the electric current associated with charge k , which is conserved,

$$\partial_\mu j^\mu = q \int_{-\infty}^{\infty} ds \partial_\mu \delta^{(4)}(x - \gamma_s) \dot{\gamma}_s^\mu = -q \int_{-\infty}^{\infty} ds \partial_s \delta^{(4)}(x - \gamma_s) = 0.$$

The self-force problem of classical electrodynamics, to which we shall return in section 2.2.3, refers to the fact that the EM field generated by (4) diverges everywhere on the world line, $\bar{\gamma} \equiv \cup_s \gamma_s$, traced by γ , rendering the Lorentz force (1) ill defined (Another troubling aspect of the self-force problem is the divergence of formally conserved quantities such as energy and momenta.)

The above unorthodox formulation of classical electrodynamics highlights its *scale covariance*, meaning that the scaled variables

$$A'(x) = \lambda^{-1} A(\lambda^{-1}x), \quad \gamma'(s) = \lambda \gamma(\lambda^{-2}s), \quad (6)$$

also solve (1)¹ and (4), *without scaling of any parameter*. Like the Poincaré symmetry, the scaling symmetry (6) admits both an active and a passive interpretation. In the active sense, it relates between different solutions of the theory, for a given labeling (unit-length convention) of space-time, from which one can read the representation under which each variable, and products thereof, transform in a scale transformation — its ‘scaling dimension’: $[x] = [\gamma] = 1$; $[s] = 2$; $[A] = [m] = -1$; $[j] = -3$ and, by definition, $[q] = 0$.

In the passive interpretation, the symmetry (6) prescribes how one must rescale A (more generally, functions defined on space-time) when relabeling (scaling the unit-length) space-time. In this sense, A can be measured in units of length, and its scaling dimension may just as well be named its length dimension, or simply dimension.

The simplicity in which scale covariance emerges in classical electrodynamics is due to the representation of a charge by a mathematical point, obviously invariant under scaling of space-time. As we shall see, achieving scale covariance with extended charges is a lot more difficult, as no dimensionful parameter may be introduced into the theory from which the charge may inherit its typical scale.

Associated with symmetry (6) is an interesting conserved ‘dilatation current’

$$\xi^\nu = p^{\nu\mu} x_\mu - \sum_{k=1}^n \int ds \delta^{(4)}(x - {}^k\gamma_s) s \, {}^k\dot{\gamma}_s^2 \, {}^k\dot{\gamma}_s^\nu, \quad (7)$$

with

$$p^{\nu\mu}(x) = \frac{1}{4} g^{\nu\mu} F^2 + F^{\nu\rho} F_\rho{}^\mu + \sum_{k=1}^n {}^k m^{\nu\mu}, \quad (8)$$

the (formally) conserved energy-momentum (e-m) tensor associated with translation covariance, and

$$m^{\nu\mu} = \int_{-\infty}^{\infty} ds \, \dot{\gamma}^\nu \dot{\gamma}^\mu \delta^{(4)}(x - \gamma_s), \quad (9)$$

¹An addition of the Lorentz-Dirac radiation reaction force, written in our convention as $\frac{2}{3}q^2 \left(\frac{\ddot{\gamma}}{\dot{\gamma}^2} - \frac{\dot{\gamma} \cdot \ddot{\gamma} \dot{\gamma}}{(\dot{\gamma}^2)^2} \right)$, still preserves the symmetry (6).

the ‘matter’ e-m tensor, formally satisfying

$$\partial_\nu m^{\nu\mu} = F^{\mu\nu} j_\nu, \quad (10)$$

$$\begin{aligned} \partial_\nu m^{\nu\mu} &= \int ds \dot{\gamma}^\nu \dot{\gamma}^\mu \partial_\nu \delta^{(4)}(x - \gamma_s) = - \int ds \dot{\gamma}^\mu \partial_s \delta^{(4)}(x - \gamma_s) \\ &= \int ds \ddot{\gamma}^\mu \delta^{(4)}(x - \gamma_s) = \int ds q F^{\mu\nu} \dot{\gamma}_\nu \delta^{(4)}(x - \gamma_s) = F^{\mu\nu} j_\nu. \end{aligned}$$

Note that the conserved dilatation charge, $\int d^3\mathbf{x} \xi^0$, depends on the choice of origin for both space-time, and the n parameterizations of ${}^k\gamma$, and is therefore difficult to interpret.

2.2 Extended Charge Dynamics

In a nutshell, the transition from classical electrodynamics to ECD, involves two modifications. The first grants the electric current (5) a nonsingular support in a way respecting all the symmetries of classical electrodynamics — scale covariance in particular. To this end we add to the representation of each charge an auxiliary complex (more generally spinor valued; see appendix B) ‘wave-function’ ${}^k\phi(x, s) : \mathbb{M} \times \mathbb{R} \mapsto \mathbb{C}$, and modify the current (5) to read

$${}^k j^\mu(x) = \int_{-\infty}^{\infty} ds \frac{iq}{2} \left[{}^k\phi(D^\mu {}^k\phi)^* - {}^k\phi^* D^\mu {}^k\phi \right] \equiv \int_{-\infty}^{\infty} ds q \operatorname{Im} {}^k\phi^* D^\mu {}^k\phi, \quad (11)$$

with

$$D_\mu = \bar{h} \partial_\mu - iq A_\mu \quad (12)$$

the gauge covariant derivative and \bar{h} some real dimensionless ‘quantum parameter’, not to be confused with \hbar . Note the similar structure of (11) and (5). In (5) it is the trace in Minkowski’s space of a singular vector-valued distribution, $\delta^{(4)}(x - {}^k\gamma_s) {}^k\dot{\gamma}_s^\mu$, generating the current, whereas in (11), the corresponding distribution is $\operatorname{Im} {}^k\phi^* D^\mu {}^k\phi$, and need not be singular. Despite this similarity between the ECD current (11) and the classical current (5), there is a striking difference between the two: the EM potential A enters the definition of the current (through D ’s dependence on it) which, in turn, depends on *all* charges. It will be demonstrated how this interdependence, along with an implicit dependence of ϕ on A , described next, leads to quantum mechanical ‘entanglement’.

Summarizing, each ECD charge is now represented by a pair $\{\phi, \gamma\}$ but, of course, ϕ is not independent of γ , as described next.

2.2.1 The ‘refined’ central ECD system

The second component of ECD is the *central ECD system* — the counterpart of the Lorentz force equation (1) — prescribing the set of permissible pairs $\{\phi, \gamma\}$ for a given A . This

system is composed of two coupled equations. The first reads

$$\begin{aligned}\phi(x, s) &= -2\pi^2 \bar{h}^2 \epsilon i \int_{-\infty}^{s-\epsilon} ds' G(x, \gamma_{s'}; s - s') \phi(\gamma_{s'}, s') \\ &\quad + 2\pi^2 \bar{h}^2 \epsilon i \int_{s+\epsilon}^{\infty} ds' G(x, \gamma_{s'}; s - s') \phi(\gamma_{s'}, s') \\ &\equiv -2\pi^2 \bar{h}^2 \epsilon i \int_{-\infty}^{\infty} ds' G(x, \gamma_{s'}; s - s') \phi(\gamma_{s'}, s') \mathcal{U}(\epsilon; s - s'),\end{aligned}\tag{13}$$

$$\text{with} \quad \mathcal{U}(\epsilon; \sigma) = \theta(\sigma - \epsilon) - \theta(-\sigma - \epsilon),$$

and the second equation is

$$\partial |\phi(\gamma_s, s)|^2 \equiv \partial |\phi(x, s)|^2 \big|_{x=\gamma_s} = 0.\tag{14}$$

Above, $G(x, x'; s)$ is the *propagator* of a proper-time Schrödinger equation (also known as a five dimensional Schrödinger equation, or Stueckelberg's equation),

$$\left[i\bar{h}\partial_s - \mathcal{H}(x) \right] G(x, x'; s) = 0, \quad \text{with } \mathcal{H} = -\frac{1}{2}D^2,\tag{15}$$

satisfying the initial condition (in the distributional sense),

$$G(x, x'; s) \xrightarrow{s \rightarrow 0} \delta^{(4)}(x - x').\tag{16}$$

Finally, ϵ is a parameter of dimension 2, ultimately taken to zero (thereby eliminating the single dimensionful parameter of ECD). Although advertised as the counterpart of the Lorentz force, it is clear that the system also applies to chargeless particles, viz. particles with a vanishing monopole.

Both equations, (13) and (14), involve a delicate $\epsilon \rightarrow 0$ limit, requiring clarifications which were not fully given in [5]. Focusing first on (13), we see that, for fixed γ and G , it is in fact an equation for a function $f^R(s) \equiv \phi(\gamma_s, s)$. Indeed, plugging an ansatz for f^R into the r.h.s. of (13), one can compute $\phi(x, s) \forall s, x$, and in particular for $x = \gamma_s$, which we call $f^L(s)$. The linear map $f^R \mapsto f^L$ (which, using $G(x', x; s) = G^*(x, x'; -s)$, can be shown to be formally self-adjoint) must therefore send f^R to itself, for (13) to have a solution. Now, the universal (viz. A -independent) $i/(2\pi\bar{h}s)^2$ divergence of $G(x, x', s)$, implies $f^R \mapsto f^R + O(\epsilon)$, so the nontrivial content of (13) is in this $O(\epsilon)$ term, which we write as ϵf^r ('r' for residue), with $f^r = O(1)$.

In [5], $\lim_{\epsilon \rightarrow 0} f^r = 0$ was implied as the content of (13). While this may turn out to be true for some specific solutions (a freely moving particle, for example), the equation should take a more relaxed form

$$\text{Im} \left(\lim_{\epsilon \rightarrow 0} f^{r*} \right) f^R = 0,\tag{17}$$

where, as usual, 'Im' is the imaginary part of the entire expression to its right.

Moving next to the second ECD equation, (14), conveniently rewritten as

$$\text{Re } \bar{h} \partial \phi(\gamma_s, s) \phi^*(\gamma_s, s) = 0, \quad (18)$$

a similar isolation of the nontrivial content exists. For further use, however, we first want to isolate the contribution of the $i/(2\pi\bar{h}s)^2$ divergence of $G(x, x', s)$ to $\phi(x, s)$, for a general x other than γ_s . To this end, we need the small- s form of the propagator G . Plugging the ansatz

$$G(x, x', s) = G_{\text{f}} e^{i\Phi(x, x', s)/\bar{h}} \quad (19)$$

into (15), with

$$G_{\text{f}}(x, x'; s) = \frac{i}{(2\pi\bar{h})^2} \frac{e^{\frac{i(x-x')^2}{2\bar{h}s}}}{s^2} \text{sign}(s), \quad (20)$$

the free propagator computed for $A \equiv 0$, and expanding Φ (not necessarily real) in powers of s , $\Phi(x, x', s) = \Phi_0(x, x') + \Phi_1(x, x')s + \dots$, higher orders of Φ_k can recursively be computed with Φ_0 alone incorporating the initial condition (16) in the form $\Phi_0(x', x') = 0$ (note the manifest gauge covariance of this scheme to any order k). For our purpose, Φ_0 is enough. A simple calculation gives the gauge covariant phase

$$\Phi_0(x, x') = q \int_{x'}^x d\xi \cdot A(\xi), \quad (21)$$

where the integral is taken along the straight path connecting x' with x . Substituting (19) into (13), and expanding the integrand around s to first order in $s' - s$: $\gamma_{s'} \sim \gamma_s + \dot{\gamma}_s(s' - s)$, $\Phi_0(x, \gamma_{s'}) \sim \Phi_0(x, \gamma_s)$, $\phi(\gamma_{s'}, s') \sim f^{\text{R}}(s)$, leads to a gauge covariant definition of the *singular part of ϕ*

$$\phi^{\text{S}}(x, s) = f^{\text{R}}(s) e^{i(\Phi_0(x, \gamma_s) + \dot{\gamma}_s \cdot \xi)/\bar{h}} \text{sinc}\left(\frac{\xi^2}{2\bar{h}\epsilon}\right) \quad (22)$$

with $\xi \equiv x - \gamma_s$. Consequently, the *residual* (or *regular*) wave-function is defined via the gauge covariant equation

$$\epsilon \phi^{\text{r}}(x, s) = \phi(x, s) - \phi^{\text{S}}(x, s). \quad (23)$$

Using $\partial \Phi_0(x, \gamma_s)|_{x=\gamma_s} = A(\gamma_s)$, we have

$$\phi^{\text{S}}(\gamma_s, s) = f^{\text{R}}(s), \quad \bar{h} \partial \phi^{\text{S}}(\gamma_s, s) = i[\dot{\gamma}_s + A(\gamma_s)] f^{\text{R}}(s), \quad (24)$$

and (18) is automatically satisfied up to an $O(\epsilon)$, gauge invariant term

$$\epsilon \text{Re } \bar{h} \partial [\phi^{\text{r}}(\gamma_s, s) \phi^{\text{S}}(\gamma_s, s)^*] = \epsilon \text{Re } D \phi^{\text{r}}(\gamma_s, s) \phi^{\text{S}}(\gamma_s, s)^*, \quad (25)$$

where the above equality follows from (24), $\phi^{\text{r}}(\gamma_s, s) = f^{\text{r}}(s)$ and (17). The refined definition of (14) is therefore

$$\lim_{\epsilon \rightarrow 0} \text{Re } D \phi^{\text{r}}(\gamma_s, s) \phi^{\text{S}}(\gamma_s, s)^* = 0. \quad (26)$$

Using the above definitions, (17) can also be written as

$$\lim_{\epsilon \rightarrow 0} \text{Im } \phi^{\text{r}}(\gamma_s, s) \phi^{\text{s}}(\gamma_s, s)^* = 0. \quad (27)$$

More insight into this refinement of the central ECD system, shall be given in the sequel. For the time being, let us just note that it is invariant under the original symmetry group of ECD. In particular, the system is invariant under

$$\phi^{\text{s}} \mapsto C \phi^{\text{s}}, \quad \phi^{\text{r}} \mapsto C \phi^{\text{r}}, \quad C \in \mathbb{C}, \quad (28)$$

under a gauge transformation

$$A \mapsto A + \partial \Lambda, \quad G(x, x', s) \mapsto G e^{i[q\Lambda(x) - q\Lambda(x')]/\hbar}, \quad \phi^{\text{s}} \mapsto \phi^{\text{s}} e^{iq\Lambda/\hbar}, \quad \phi^{\text{r}} \mapsto \phi^{\text{r}} e^{iq\Lambda/\hbar}, \quad (29)$$

and under scaling of space-time

$$\begin{aligned} A(x) &\mapsto \lambda^{-1} A(\lambda^{-1} x), \quad \epsilon \mapsto \lambda^2 \epsilon, \quad \gamma(s) \mapsto \lambda \gamma(\lambda^{-2} s), \\ \phi^{\text{s}}(x, s) &\mapsto \lambda^{-2} \phi^{\text{s}}(\lambda^{-1} x, \lambda^{-2} s), \quad \phi^{\text{r}}(x, s) \mapsto \lambda^{-2} \phi^{\text{r}}(\lambda^{-1} x, \lambda^{-2} s), \end{aligned} \quad (30)$$

directly following from the transformation of the propagator under scaling

$$A(x) \mapsto \lambda^{-1} A(\lambda^{-1} x) \Rightarrow G(x, x'; s) \mapsto \lambda^{-4} G(\lambda^{-1} x, \lambda^{-1} x'; \lambda^{-2} s).$$

Regarding this last symmetry, two points should be noted. First, for a finite ϵ it relates between solutions of *different* theories, indexed by different values of ϵ . It is only because ϵ is ultimately eliminated from all results, via an $\epsilon \rightarrow 0$ limit, that scaling can be considered a symmetry of ECD. The second point concerns the scaling dimension, -2 , of ϕ^{s} and ϕ^{r} . By the symmetry (28), that dimension can be arbitrarily chosen. However, the central ECD system is but a part of the ECD formalism, which dictates this special choice of dimension to comply with scale covariance (see next section).

2.2.2 Regularized currents

The $\epsilon \rightarrow 0$ limit of the current (11), as also the limits of all other currents in ECD, vanishes everywhere (except on the world line, $\bar{\gamma} \equiv \cup_s \gamma_s$, traced by γ , where it is finite), trivializing ECD. To correct this situation, two steps are taken. Utilizing the symmetry (28), we first rescale $\phi \mapsto \epsilon^{-1} \phi$, granting j in (11) a nonsingular support. As we show next, however, the resultant current diverges everywhere in the limit $\epsilon \rightarrow 0$. To fix this new problem, we substitute $\phi \mapsto \epsilon^{-1} \phi^{\text{s}} + \phi^{\text{r}}$ in (11), and note that this divergence, as well as those of all other ECD currents, can be traced to gauge invariant contributions of bilinears in ϕ^{s} , which in this case read

$$\int_{-\infty}^{\infty} ds \, q \, \text{Im} \frac{\phi^{\text{s}*}}{\epsilon} D^{\mu} \frac{\phi^{\text{s}}}{\epsilon} \equiv j^{\text{s}}. \quad (31)$$

Indeed, by (22) we get

$$j^{\text{s}}(x) = \frac{q}{\hbar} \int ds \, (\dot{\gamma}_s - qA(x) + \partial_x \Phi(x, \gamma_s)) |f^{\text{R}}(s)|^2 \frac{1}{\epsilon^2} \text{sinc}^2 \left(\frac{(x - \gamma_s)^2}{2\hbar\epsilon} \right). \quad (32)$$

Using (in the distributional sense) $\epsilon^{-1}\text{sinc}^2(\epsilon^{-1}y) \longrightarrow \pi\delta(y)$ for $\epsilon \rightarrow 0$, we see that j^s contains an ϵ^{-1} term in its ϵ -expansion. Taking further into account the finite width of $\epsilon^{-1}\text{sinc}^2(\epsilon^{-1}y)$ (as oppose to a delta distribution) and its evenness, it can be shown that the next higher power in the expansion is ϵ^1 . This leads to the definition of the *regular current*, j^r — a gauge invariant expression defined as the free coefficient in the ϵ -expansion of j , or equivalently,

$$j^r = \lim_{\epsilon \rightarrow 0} (j - j^s) . \quad (33)$$

This regular current is the electric current ultimately associated with an ECD charge, entering as a source into Maxwell's equations (4). By (30), j^r has dimension -3 , consistent with the scaling dimension of A , namely, (4) is invariant under

$$A(x) \mapsto \lambda^{-1}A(\lambda^{-1}x) , \quad j^r(x) \mapsto \lambda^{-3}j^r(\lambda^{-1}x) , \quad (34)$$

establishing the scale covariance of ECD. Finally, we note that for $A \equiv 0$ and a freely moving γ ($\gamma_s = us$), j^r vanishes, as it must on self consistency grounds.

In appendix A we prove that the regular current, (33), is conserved for $x \notin \bar{\gamma}$. The conservation of a current defined on $\mathbb{M}/\bar{\gamma}$, however, does not imply the time independence of the associated charge $Q = \int d^3\mathbf{x} j^{r0}(x^0, \mathbf{x})$, due to a possible ‘leakage’ of charge into a sink of j^r on $\bar{\gamma}$ or ‘emergence’ of charge from a source thereon. Remarkably, the refined ECD equation (27), turns out to be exactly the condition guaranteeing that no such leakage occurs. Likewise, the second refined ECD equation, (26), guarantees that no energy or momenta leak into sinks of the conserved energy-momentum tensor on $\cup_k {}^k\bar{\gamma}$. It is therefore natural to add to the central ECD system the proviso that the electric charge of each particle, as well as the collective e-m of the system, do not ‘leak to infinity’ (although it is possible that at least the first condition is automatically satisfied).

Carefully applying Noether's theorem, both charge and energy-momentum conservation, can be shown to follow from continuous symmetries of ECD, (see appendix A). The converse, however, is not true, namely, not every continuous symmetry of ECD leads to a conservation law, due to a possible leakage of the corresponding charge to sinks on ${}^k\bar{\gamma}$. The counterparts of the ‘mass-squared current’, (2), associated with s -translation invariance, as well as the counterpart of (7) corresponding to scale covariance, fall into this category (see appendix A).

2.2.3 The self consistent potential

As in classical electrodynamics, so also in ECD, the EM potential, A , must satisfy a self consistent ‘loop’:

- (a) Start with A and n pairs $\{{}^k\phi, {}^k\gamma_s\}$ satisfying the central ECD system (26), (27) (or the Lorentz force equation, (1), in classical electrodynamics);
- (b) From these, using (11) (or (5) in classical electrodynamics), compute n ${}^k j^r$'s, plug them into the r.h.s. of (4) and, finally,
- (c) verify that the l.h.s. agrees with the original A .

To the above loop one should add the proviso that the electric and mass-squared charges

of each particle, as well as the total energy and momentum of the system, must be finite. As shown in appendix A, this proviso involves at most the asymptotic behavior of A and ϕ away from γ , and not the classical self-energy divergence which is automatically eliminated.

The common loop notwithstanding, two important differences should be noted. First, in classical electrodynamics, the loop is only formal, due to the self-force problem — the ill defined Lorentz self-force at the position of a point-charge. Resolving the self-force problem calls for extensions/deformations of classical electrodynamics which, while eliminating the original problem, also introduce drastic changes to other, desirable, features of it². An even greater shortcoming is the fact that those extensions/deformations of classical electrodynamics do not survive experimental tests when applied to small scales. This is the domain of QM, along with its distinct ontology. In ECD, on the other hand, only $\phi^r(x, s)$ needs to be differentiable on $\bar{\gamma}$. This condition easily tolerates discontinuities of the EM field on $\bar{\gamma}$ (which, in fact, exist) freeing ECD, as is, from the self-force problem.

The second difference in the role played by the above loop is that, in ECD, the very existence of an ECD charge is due to a solution for the loop. That is, a non vanishing A must be found even for a single static charge in an otherwise void universe — different such solutions naturally corresponding to different elementary particles. This is a nontrivial requirement, possibly leading to constraints on the nature of fundamental ECD charges. Charge quantization is one such possibility, as the magnitude of the total charge of a solution

²Attempts to resolve the classical self-force problem follow two distinct approaches. In the first, it is postulated that the field produced by a charge does not act on the charge itself, thereby allowing to preserve the point structure of elementary charges, along with their scale covariance (a point remains a point following the scaling of space-time). The Abraham-Lorentz-Dirac equation, or action-at-a-distance electrodynamics, [8], are typical examples of this approach, both carrying a heavy price-tag in the form of expressions for the (conserved) energy and momenta which are radically different from the (ill-defined but rather intuitive) corresponding expressions in the usual formulation of electrodynamics (in fact, in the case of the Abraham-Lorentz-Dirac equation, it is not even clear what is conserved in a system of interacting charges; no Lagrangian for such a system is known).

In the second approach, the singular self-force is resolved by ‘extending’ the charge so as to associate with it a nonsingular current producing a smooth self field. Prominent examples include Lorentz’s early attempt to regard the electron as a small uniformly charged sphere, and Feynman’s covariant regularization of the self Lienard-Wiechert potential [4]. Done in the framework of action-at-a-distance electrodynamics, Feynman directly regularizes the Lienard-Wiechert potential rather than deriving it from a regular current. However, his method can be readily adopted to the regularization of a current. Defining

$$j(x) = \partial^2 \int_{-\infty}^{\infty} ds \frac{1}{r^2} f \left[\frac{(x - \gamma_s)^2}{r^2} \right] \dot{\gamma}_s$$

for some integrable function f , we note, with Feynman, that j is everywhere conserved and negligible for distances from the charge much greater than r (a covariant alternative, more in the spirit of what is done in this paper, is, rather than operating with the d’Alembertian on the integral, taking its derivative $\partial/\partial(r^2)$). This latter approach, however, introduces a privileged scale (e.g. r in the above example) into otherwise scale covariant electrodynamics, dramatically diminishing its symmetry group, and also introduces infinitely many tunable constants — the function f — into single-parameter classical electrodynamics. Taking the limit $r \rightarrow 0$ introduces into the dynamics of γ a term of the form $C\ddot{\gamma}$ with C diverging in that limit, trivializing the dynamics (the charges cannot move due to an infinite effective mass). To get a nontrivial theory, one must then ‘renormalize the mass’ — a rather contrived procedure.

is invariant under the full symmetry group of ECD. Another possibility is that, as in other eigenvalue problems, only for certain values of the ECD parameters, viz. \bar{h} , q , and g (for spin- $\frac{1}{2}$ ECD), does there exist a solution.

The self consistent loop is further responsible for the emergence of a privileged scale (e.g. the size/mass of the charge) in a scale covariant theory, containing no dimensionful parameter. Of course, the scale selected is but one of the infinite possibilities, corresponding to different λ in the scaling transformation (30).

A closely related aspect of the loop concerns the fact that A is generated by the collective current of *all* the particles. As A enters the expressions of all the currents associated with an ECD charge (both explicitly and implicitly via ϕ), a dense cloud of particles must therefore be strongly entangled by this constraint, as apparently manifested in low temperature experiments. Moreover, even attributes of individual particles, such as the mass or charge, are, in principle, entangled. (This may explain why a continuum of scaled interacting atoms are never seen — a self consistent A may not exist for the combined system. We shall have a lot more to say about role of the consistency loop with regard to entanglement of remote particles.)

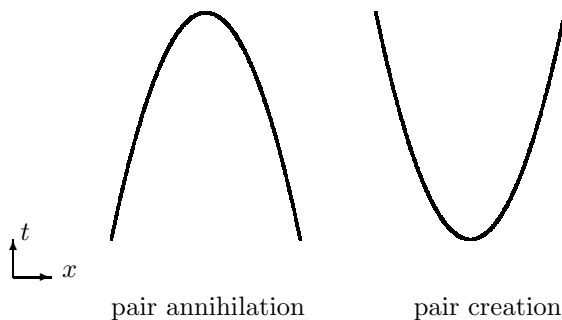
2.2.4 Antiparticles

ECD can be shown to be invariant under a ‘CPT’ transformation

$$\begin{aligned} A(x) &\mapsto -A(-x), \quad \gamma(s) \mapsto -\gamma(-s) \quad \phi(x, s) \mapsto \phi^*(-x, -s) \\ &\Rightarrow j^r(x) \mapsto -j^r(-x). \end{aligned} \tag{35}$$

In fact, scalar ECD, as well as classical electrodynamics³, enjoys an even larger symmetry group, C: $A(x) \mapsto -A(x)$, $j^r(x) \mapsto -j^r(x)$; and PT: $A(x) \mapsto A(-x)$, $j^r(x) \mapsto j^r(-x)$. However, the spin- $\frac{1}{2}$ ECD, presented in appendix B, enjoys the CPT symmetry only. This symmetry has some remarkable consequences. First, it implies that our naive notion of time-reversal — ‘running the movies backward’ — is not a symmetry of micro-physics. Secondly, it predicts the existence of an antiparticle for each particle (viz., a bound solution of one or more elementary ECD charges) of opposite charge and equal self-energy. Pair creation/annihilation may then have a simple geometrical interpretation when γ ‘reverses its direction in time’ (see picture).

³Maxwell’s equations and the Lorentz force are also symmetric under under T: $\gamma(t) \mapsto \gamma(-t)$, $\mathbf{E}(\mathbf{x}, t) \mapsto \mathbf{E}(\mathbf{x}, -t)$, $\mathbf{B}(\mathbf{x}, t) \mapsto -\mathbf{B}(\mathbf{x}, -t)$, and under P: $\gamma(t) \mapsto -\gamma(t)$, $\mathbf{E}(\mathbf{x}, t) \mapsto \mathbf{E}(-\mathbf{x}, t)$, $\mathbf{B}(\mathbf{x}, t) \mapsto -\mathbf{B}(-\mathbf{x}, t)$. However, if one includes in the definition of classical electrodynamics, a definite Green’s function, other than the half-advanced-plus-half-retarded-Lienard-Wiechert-potential, then T is no longer a symmetry.



As a particle and its antiparticle have opposite signs for both their electric charges, and their mass-squared charges (expression (95), the counterpart of the classical (2)), such annihilation/creation scenarios respect the conservation laws of both electric and mass-squared charges. The energy of a particle, however, equals that of its antiparticle. In such annihilation processes, EM radiation must be released in order to respect energy conservation. It may even be possible for the particle and its antiparticle to belong to distinct γ 's. This, however, is more speculative, as the two may also form a bound neutral state, waiting dormant to be re-separated by, say, a strong EM pulse.

2.2.5 The semiclassical approximation of the central ECD system

The semiclassical, or small \hbar analysis of the central ECD system, is facilitated by the leading order in the \hbar expansion of the propagator, G , known as the *semiclassical propagator* ⁴

$$G_{\text{sc}}(x, x'; s) = \frac{i \operatorname{sign}(s)}{(2\pi\hbar)^2} \sum_{\beta} \mathcal{F}_{\beta}(x, x'; s) e^{iI_{\beta}(x, x'; s)/\hbar}. \quad (36)$$

Here, β runs over the different classical paths, viz. paths solving (1) for the fixed A , such that $\beta(0) = x'$ and $\beta(s) = x$; I_{β} is the corresponding action of the path,

$$I_{\beta} = \int_0^s d\sigma \frac{1}{2} \dot{\beta}_{\sigma}^2 + qA(\beta_{\sigma}) \cdot \dot{\beta}_{\sigma}, \quad (37)$$

and \mathcal{F} — the so called Van-Vleck determinant — is the gauge-invariant classical quantity, given by the determinant

$$\mathcal{F}(x, x'; s) = \left| -\partial_{x_{\mu}} \partial_{x'_{\nu}} I_{\beta}(x, x'; s) \right|^{1/2}. \quad (38)$$

Let us next show that to leading order in \hbar , the refined central ECD system is solved by any classical γ (in the given EM potential A), and by a corresponding ansatz of the form

$$f^{\text{R}}(s') = C e^{iI_{\gamma}(\gamma_{s'}, \gamma_0, s')/\hbar}, \quad (39)$$

⁴The nature of the approximation involved in the use of the semiclassical propagator can be read from the path integral representation of the propagator. The classical paths dominate that representation and the semiclassical approximation amounts to mis-weighting paths that deviate significantly from classical paths. These, however, enter with a nearly random phase anyway.

where $C \in \mathbb{C}$ is arbitrary. To the extent that the semiclassical approximation is valid, i.e. that \bar{h} is sufficiently small, this explicitly proves that any significant deviation from a classical path is entirely due to the self field.

Substituting in (13), $G \mapsto G_{\text{sc}}$, $x' \mapsto \gamma_{s'}$ and $x \mapsto \gamma_s$, we first note that one of the β 's, appearing in G_{sc} , connecting $\gamma_{s'}$ with γ_s , must coincide with γ (as γ is a classical path in A , connecting $\gamma_{s'}$ with γ_s). There are, in general, other one-parameter families of *indirect paths*, $s'\beta(\sigma)$, parametrized by s' , connecting $\gamma(s')$ with $\gamma(s)$ not via $\bar{\gamma}$ (e.g. bouncing off of a remote potential). Focusing first on this direct contribution, and using

$$I_\gamma(\gamma_s, \gamma_{s'}, s - s') I_\gamma(\gamma_{s'}, \gamma_0, s') = I_\gamma(\gamma_s, \gamma_0, s) \quad (40)$$

we get

$$\begin{aligned} \phi(\gamma_s, s) &= \frac{\epsilon C}{2} e^{iI_\gamma(\gamma_s, \gamma_0, s)/\bar{h}} \int_{-\infty}^{\infty} ds' \mathcal{F}_\gamma(\gamma_s, \gamma_{s'}; s - s') \text{sign}(s - s') \mathcal{U}(\epsilon; s - s') \\ &\Rightarrow \phi^r(\gamma_s, s) = \frac{C}{2} e^{iI_\gamma(\gamma_s, \gamma_0, s)/\bar{h}} \left[R(s, \epsilon) - \frac{2}{\epsilon} \right], \end{aligned} \quad (41)$$

with

$$R(s) = \int_{-\infty}^{\infty} ds' \mathcal{F}_\gamma(\gamma_s, \gamma_{s'}; s - s') \text{sign}(s - s') \mathcal{U}(\epsilon; s - s') \quad (42)$$

some real functional of the EM field and its first derivative (its local neighborhood in an exact analysis) on $\bar{\gamma}$, such that $\lim_{\epsilon \rightarrow 0} [R(s, \epsilon) - 2/\epsilon]$ is finite, implying that (27) is satisfied.

Moving next to the second refined ECD equation, (26), and pushing ∂ into the integral in (13),

$$\begin{aligned} \bar{h} \partial \phi(\gamma_s, s) &= \frac{\epsilon C}{2} e^{iI_\gamma(\gamma_s, \gamma_0, s)/\bar{h}} \int_{-\infty}^{\infty} ds' \left[i \partial_x I_\gamma(x, \gamma_{s'}; s - s') \Big|_{x=\gamma_s} \mathcal{F}_\gamma(\gamma_s, \gamma_{s'}; s - s') \right. \\ &\quad \left. + \bar{h} \partial_x \mathcal{F}_\gamma(x, \gamma_{s'}; s - s') \Big|_{x=\gamma_s} \right] \text{sign}(s - s') \mathcal{U}(\epsilon; s - s'). \end{aligned} \quad (43)$$

The second term in (43) can be neglected for small \bar{h} . Using a relativistic variant of the Hamilton-Jacobi theory (see appendix B in [5]), we can write

$$\partial_x I_\gamma(\gamma_s, \gamma_{s'}, s - s') = p(s) \equiv \dot{\gamma}_s + qA(\gamma_s)$$

which is independent of s' . The first term in (43) therefore gives

$$\begin{aligned} \bar{h} \partial \phi(\gamma_s, s) &= ip(s) \phi(\gamma_s, s) \Rightarrow \bar{h} \partial \phi^r(\gamma_s, s) = ip(s) \phi^r(\gamma_s, s) \\ &\Rightarrow \lim_{\epsilon \rightarrow 0} \text{Re } D \phi^r(\gamma_s, s) f^{\text{R}*}(s) = -\dot{\gamma}_s \lim_{\epsilon \rightarrow 0} \text{Im } \phi^r(\gamma_s, s) f^{\text{R}*}(s), \end{aligned} \quad (44)$$

which vanishes by (27), hence (26) is satisfied.

We return now to the contribution of the indirect-paths, β , in the sum over classical paths, appearing in the definition of G_{sc} . The phase of the corresponding integrand in (13) reads

$$\bar{h}^{-1} I_\beta(\gamma_s, \gamma_{s'}, s - s') I_\gamma(\gamma_{s'}, \gamma_0, s') . \quad (45)$$

As distinct $s'\beta$ and $s'\gamma$ see different potentials, and do not lie on the same mass-shell, (45) does depend on s' — the s' -independence, manifested in (40), is a privilege of $\beta = \gamma$. Combined with the smallness of \hbar , the contributions of the indirect paths are therefore suppressed by the strong oscillation of the phase (45).

3 Qualitative discussion of ECD

The ECD formalism presented in the previous section has a rather unusual structure. Explicit solutions, relevant to physically interesting cases, are difficult to solve, apparently necessitating an extensive use of numerical calculations. However, the stage is completely set for such detailed analysis. Isolated, self consistent ECD solutions or bound states of any number of them, can be sought, possibly (and most desirably in the author's opinion) showing that all elementary particles are just different solutions of the same set of equations (significantly reducing the number of tunable constants). The effective mass and binding energies of such solutions can be computed using the expression for the energy momentum tensor derived in the appendices; detailed internal structures of such particles can be analyzed, possibly suggesting novel methods of ‘cracking’ (or fusing) subatomic particles. In short, one can potentially have a clear, scale covariant ontology, based on interacting ECD particles alone, rendering additional forces and particles superfluous. ECD, then, is clearly not merely an interpretation of QM, but rather a complementary theory with independent testable predictions.

To motivate such an endeavor, this section argues the case for the compatibility of ECD with current, well tested, physics. ECD, essentially retaining the ontology of classical electrodynamics, is apparently susceptible to the same objections and ‘no go theorems’ standing in the way of other hidden variables models. In particular, since the EM field is just the classical Maxwellian field, one may rightfully wonder where does the ‘photon’ come from? We shall demonstrate to the contrary, that in a wide range of cases in which the statistical predictions of QM clearly cannot be realized by an ensemble of classical solutions, the unique features of ECD could render possible such a realization by an ensemble of ECD solutions, resulting in a rather prosaic ‘demystification’ of QM — the photon included.

For simplicity, only the scalar case is analyzed. The spin of an ECD particle merely labels different ways of covariantly obtaining extended currents — ordinary currents, transforming as four-vectors. An example of spin- $\frac{1}{2}$ ECD is covered in appendix B.

3.1 Single-body ECD

Single-body ECD deals with the ECD equations of a single particle in the presence of an external EM potential, where the use of the term ‘particle’, rather than charge, reflects the possibility for an elementary ECD solution to have a vanishing monopole. Specifically, we assume the existence of an external potential, A_{ext} , satisfying Maxwell's equations (4) for some fixed current, j_{ext} , generated by the rest of the particles in the universe, assumed independent of the particle in question. This is clearly a simplification of the real situation

to which we return in section 3.2, dealing with many-body ECD. Next, we ‘feed’ $A_{\text{ext}} + A_{\text{sel}}$ into the self consistent loop of section 2.2.3, solving ϕ in the presence of $A_{\text{ext}} + A_{\text{sel}}$, and close the loop by requiring from the self potential to satisfy

$$\square A_{\text{sel}}^\mu - \partial^\mu (\partial \cdot A_{\text{sel}}) = j^{\text{r} \mu}, \quad (46)$$

with j^{r} computed from ϕ and the combined potential $A_{\text{ext}} + A_{\text{sel}}$.

Modulo the self-force problem, the corresponding classical theory trivializes to finding solutions of (1) in the the presence of F_{ext} . While the exact ECD solution, explicitly incorporating self interaction effects, is much more complicated, its energy-momentum balance follows quite similar lines. In appendix A, a relation, (90), is derived, formally identical to its classical counterpart (10), relating the energy momentum tensor, m , associated with a particle, to its conserved electric current j (omitting the regularization label r)

$$\partial_\nu m^{\nu\mu} = F^{\mu\nu} j_\nu \equiv (F_{\text{ext}}^{\mu\nu} + F_{\text{sel}}^{\mu\nu}) j_\nu, \quad (47)$$

where F_{sel} is the self-field derived from A_{sel} via (46). Let $\Sigma(s)$ be a one-parameter family of non intersecting time-like surfaces, each intersecting $\bar{\gamma}$ at γ_s , C a four-cylinder containing $\bar{\gamma}$ and $p^\mu(s)$ the corresponding four-momenta

$$p^\mu = \int_{\Sigma(s) \cap C} d\Sigma_\nu m^{\nu\mu}, \quad (48)$$

where $d\Sigma$ is the Lorentz covariant directed surface element, orthogonal to $\Sigma(s)$. Let also $C(s, \delta) \in C$ be the volume enclosed between $\Sigma(s)$ and $\Sigma(s + \delta)$, and $T(s, \delta)$ its space-like boundary (see figure 1 for a 1 + 1 counterpart).

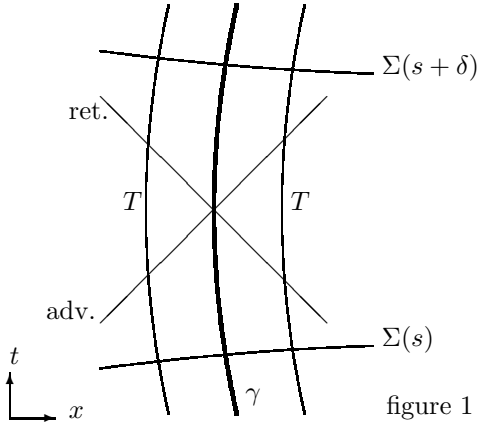


figure 1

Integrating (47) over $C(s, \delta)$, and applying Stoke’s theorem to the l.h.s., we get

$$p^\mu(s + \delta) - p^\mu(s) + \int_T dT_\nu m^{\nu\mu} = \int_{C(s, \delta)} d^4x (F_{\text{ext}}^{\mu\nu} + F_{\text{sel}}^{\mu\nu}) j_\nu. \quad (49)$$

with dT the outward pointing directed surface element on T . For a point charge with m and j given by (10) and (5) resp., the term $\int_T dT_\nu m^{\nu\mu}$ vanishes, $p = \dot{\gamma}$, and upon taking the limit

$\delta \rightarrow 0$ and dividing by δ , (49) formally becomes just the Lorentz force equation (1) with $F = F_{\text{ext}} + F_{\text{sel}}$. As noted before, however, the self force is ill-defined in the classical case (hence the reservation implied in ‘formally’). In moving from a singular electric current to an extended one, the first benefit is that now the self-force appearing in (49) is well defined. For a static charge, for example, the only non vanishing component of the electric current is $j^0(\mathbf{x})$ from which the purely electrostatic F_{sel} inherits its spherical symmetry, leading to a vanishing self force. The simplest complication of the static case, then, is when the currents retain an approximate spherical symmetry (in the rest frame of γ) and F_{sel} is approximately a non radiating spherical electrostatic field, contributing a negligible self-force only. Under this assumption we can write $p = \alpha \dot{\gamma}$ with α some positive constant, and

$$\lim_{\delta \rightarrow 0} \delta^{-1} \int_{C(s, \delta)} d^4x F_{\text{ext}}^{\mu\nu} j_\nu = Q \langle F^{\mu\nu} \rangle_s \dot{\gamma}_\nu, \quad (50)$$

with $Q = \int_{\Sigma(s)} d\Sigma \cdot j$ the s -independent electric charge (here we assume that $\Sigma(s) \cap C$ supports the lion’s share of the charge) and $\langle F^{\mu\nu} \rangle_s$ is the average field in $\Sigma(s)$, weighted by the normalized charge density. (The above equalities are most conveniently established in the rest-frame of γ where j^0 and m^{00} are the only non vanishing components, $\Sigma(s)$ is taken to be $x^0 = \gamma_s^0$ three-space, the Lorentz force density is purely electrostatic, and $dT_0 = 0 \Rightarrow dT_\nu m^{\nu\mu} = 0$.) Equation (49) then leads to

$$\alpha \ddot{\gamma}^\mu = Q \langle F^{\mu\nu} \rangle_s \dot{\gamma}_\nu, \quad (51)$$

and the constant α is identified with $\sqrt{p^2/\dot{\gamma}^2}$, where p^2 is the Lorentz invariant rest-energy of the charge. We therefore reach the important conclusion: *Whenever an ECD charge maintains an approximate spherical symmetry, its dynamics must be classical.*

It is instructive to compare the above treatment of an ECD charge with Lorentz’s modeling of the electron as a rigid, uniformly charged sphere, enabling him to obtain a well defined expression for the self-force, without going through a fishy mass-renormalization procedure (as in later treatments, preserving the point structure of the charge). As a relativistic rigid extended body is a meaningless concept, Lorentz’s sphere model is valid at most for a sufficiently uniform motion — the larger the sphere, the more uniform the motion must be. A rapidly varying external field on the scale of the sphere therefore signals the breakdown of Lorentz’s self-force analysis. Likewise, a non uniformly moving ECD particle cannot maintain an exactly spherical charge distribution in the rest frame of every point along γ , and a rapidly varying γ on the scale of the ball holding the lion’s share of the charge, dubbed the *core*, needs not even resemble a classical path. In this respect, ECD can be seen as a fully covariant extension of Lorentz’s analysis of the self-force. It is argued below that, in principle, all of QM can be traced to the breakdown of the spherical core approximation.

Assuming ECD indeed governs the microscopic world, the above spherical core model explains at once the reductions of the QED Klein-Nishina formula for the cross section in Compton scattering, to the classical Thompson formula, at wavelengths greatly exceeding the electron’s Compton length, and sets the Compton length as the order of magnitude of the core. Indeed the Thompson formula is obtained by simple averaging over the radiation

produced by point charges oscillating in an external plane wave. For wavelengths much longer than the scale of the core, we have in (51) $\langle F \rangle_s \approx F(\gamma_s)$, and point dynamics is reproduced. The spherical core approximation further accounts for another conspicuous coincidence — the agreement between the nonrelativistic classical and quantum cross sections for Coulomb scattering, both given by the \hbar -independent Rutherford formula⁵. The fact that the Coulomb potential (being an electrostatic potential) is a harmonic function, implies that the potential of a spherically symmetric core in it, equals that of the center of the core, viz. that of γ , hence no finite-core-size corrections to point dynamics are observed for this special potential.

3.1.1 The breakdown of the spherical core

Equation (47) and its integral form (49), somewhat artificially divide the change in the momentum of a particle into a work of the Lorentz force, plus a ‘radiative’ contribution, $\int_T dT_\nu m^{\nu\mu}$, of the associated e-m density m . A more symmetric treatment of ‘matter’ and the EM field is provided by the conservation of the ECD counterpart of (8), $p = \Theta + \sum_k {}^k m$ (see appendix A.1) where Θ is the canonical EM energy-momentum tensor (93). Applying Stoke’s theorem to $\partial p = 0$, and using the same construction as in figure 1, we get

$$p^\mu(s + \delta) - p^\mu(s) = - \int_T dT_\nu p^{\nu\mu}, \quad (52)$$

with

$$p^\mu = \int_{\Sigma(s) \cap C} d\Sigma_\nu p^{\nu\mu}, \quad (53)$$

the total four-momentum content of $\Sigma(s) \cap C$. Although $p^{\mu\nu}$ is due to *all* particles in the system, in the vicinity of a sufficiently isolated particle k' , p is dominated by ${}^{k'} m$ and the self field generated by ${}^{k'} j$. This all leads to the conclusion that the conservation of energy and momentum associated with an isolated particle (EM e-m included) can only be breached by an energy-momentum flux penetrating T . This flux is composed of the classical Poynting vector, plus a ‘quantum’ piece associated with ${}^{k'} m$. This ‘electro-weak’ division, nonetheless, is entirely artificial, as ${}^{k'} m$ also depends on A both explicitly and implicitly (via ϕ). Moreover, whenever the core breaks down, producing such matter e-m flux over T , a corresponding electric flux also forms, which locally modifies the Poynting flux (note that this ‘radiative component’ of j may be negligible in terms of charge capacity, and still generate a strong EM field if it strongly fluctuates).

Summarizing our findings regarding a sufficiently isolated particle, the integral of the e-m flux, $\int_T dT_\nu p^{\nu\mu}$, does not vanish only if γ is non uniform. This, of course, is a standard result of classical electrodynamics, the quantification of which leads to the celebrated Lorentz-Dirac equation (but not before ‘renormalizing’ the mass of the charge — a rather contrived procedure whose sole motivation is to render the result non trivial), and can be directly

⁵The author thanks Prof. Shimon Levit for sharing with him a long time ago his wonderment over this point.

derived from the the expression for the Lienard-Wiechert potential generated by a moving charge

$$A_{\text{adv}}^{\text{ret}}(x) = q \int ds \delta[(x - \gamma_s)^2] \dot{\gamma}_s \theta(x^0 \mp \gamma_s^0) . \quad (54)$$

The advanced and retarded potentials are the traces of densities, $\delta[(x - \gamma_s)^2] \dot{\gamma}_s \theta(x^0 \mp \gamma_s^0)$, supported on the light-cone of γ_s (schematically depicted by adv. and ret. in figure 1). These fields are then further processed, yielding a Θ which can easily be shown to contain a radiative component (viz. dropping as r^{-2} from the charge) at a point x only if γ is non uniformly moving in the neighborhood of points lying on the intersections of the light-cone of x with $\bar{\gamma}$.

The ECD counterpart of (54) can be expected to read

$$A_{\text{adv}}^{\text{ret}}(x) = q \int ds \left[\int d^4y \delta[(x - y)^2] \theta(x^0 \mp y^0) \lim_{\epsilon \rightarrow 0} J^r(s, y) \right] , \quad (55)$$

with $J^r = \text{Im } \phi^* D \phi$ removed of bilinears in ϕ^s , and the $\epsilon \rightarrow 0$ limit is to be understood in the distributional sense. Indeed, A appearing in (55) solves Maxwell's equations (4) with j^r as a source, and upon plugging in (55) the corresponding classical density $J^r(s, y) \mapsto \delta^{(4)}(y - \gamma_s) \dot{\gamma}_s$, (54) is reproduced. Equation (55), however, ignores a crucial differences between the above densities. Unlike its classical counterpart, J^r depends on A . Equation (55), unlike (54), is therefore not a prescription for $A_{\text{adv}}^{\text{ret}}$ but rather an equation for it. A solution in which only A_{adv} or A_{ret} enter J^r may not (and probably does not) exist. The ‘correct’ radiation field, containing both advanced and retarded components, so to speak, is therefore dictated by the specifics of the radiation process. In classical electrodynamics, on the other hand, a solution of Maxwell's equation (4) is defined only up to a solution of the homogeneous equation $\partial_\nu F^{\nu\mu} = 0$. For this reason, the motivation for the (almost universal) choice of the retarded potential is not in the equations proper, but rather in the desire to conform with observations concerning *large scale* radiation phenomena, involving huge numbers of particles (See more in section 3.2.2). We are led, then, to the important conclusion that in ECD, advanced EM flux, combined with a corresponding advanced mechanical flux associated with m , must be considered. We shall see in 3.2.1 apparent experimental signatures left by such advanced effects.

3.1.2 The aether and its manifestations

The picture emerging from the previous section is that of a single conserved e-m field, $p^{\mu\nu}(x)$, organically integrating, or fusing, radiation and matter. The distribution of p in \mathbb{M} is highly nonuniform, with the lion's share of the charges concentrated in small three-cylinders centered around $k\bar{\gamma}$ (and integrably singular there; see appendix A). Time-like cross sections of these tubes were dubbed ‘cores’ of particles. Amidst the densely charged three-cylinders resides a (mostly) weak, locally conserved p , smoothly merging with the dense cylinders. This inter-particle p which, in the absence of a better name shall be referred to as the *aether*, cannot be decisively attributed to any single particle as A , entering every term in

p , is generated by the combined current of *all* the particles. Nevertheless, as we have seen, its dynamics at a point is strongly correlated with that of neighboring particles.

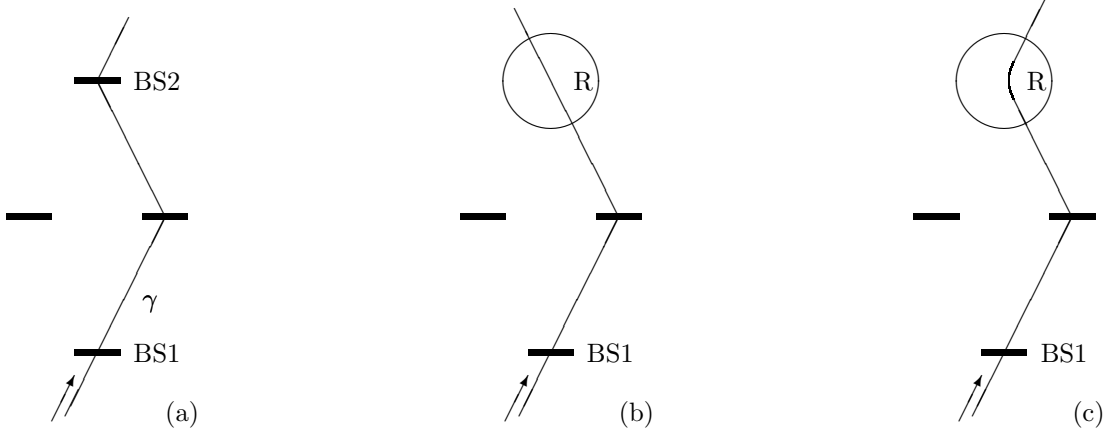
Consider now a scattering experiment in which a neutral particle passes through a small aperture in a screen. As only the collective p — cores plus aether — is conserved, there is a mutual influence between the particle and the (cores composing the) screen, mediated by the aether. This interaction, however, is extremely small in ordinary laboratory experiments, and it is only via the amplification brought about by the huge distance between the aperture (more generally — a scatterer) and the detection screen, that minute corrections to the classical cross section are detectable. One can also understand why attempts to measure the position of the particle during its flight, by shining light on it, destroy the delicate interference pattern. This is because the external EM field applied to the particle, interacts with the core. This is essentially a classical interaction, coming with a variability which greatly surpasses the feeble quantum corrections to classical paths, responsible for the fringes in the first place.

It is conjectured that gravity is yet another manifestation of such ether mediated interaction between remote cores, amplified this time by huge numbers/mass rather than by a huge distance. In particular, as Θ is part of the aether, the bending of the trajectory of an EM wave-packet in a gravitational field, can be seen as a perturbation to a nonuniform static aether configuration (just like mechanical waves bend in a nonuniform medium). In the context of ECD, therefore, gravitational theories such as general relativity (GR), only play a role similar to that of QM, as an effective statistical theory, respecting all the symmetries of ECD, but applicable to totally different experimental settings.

Let us briefly see what such a statistical theory must look like. Scale covariance naturally suggests we represent the cores by mathematical points, obviously invariant under scaling of space-time. This, as we know, creates a self force problem similar to that plaguing classical electrodynamics. GR, therefore, cannot be considered a completely satisfactory theory and, as we saw in the case of ECD, the resolution of this problem may result in radical changes to GR, not merely minute correction. Self force problem aside, we can appreciate why GR is a reasonable candidate. The e-m tensor derived from the metric, is locally conserved, transforming in a scale transformation as p , and carries ‘e-m waves’ generated by non uniformly moving bodies — much like p .

3.1.3 Interferometers

In the previous section we mentioned two ways of amplifying the small aether-induced deviations of the cores from classical paths: Huge distances and huge numbers/mass. A distinct third way, implemented in neutron interferometers, relies on the ability of chaotic systems to amplify small perturbations. In a Mach-Zehnder configuration, (a), the beam-splitters (BS) and mirrors are crystals of macroscopic thickness, forming a huge lattice of scatterers in which a particle undergoes multiple scatterings before exiting.



Even at the classical level, the dynamics in such a maze is highly chaotic, meaning, in particular, that the standard procedure of averaging over the impact parameter in order to obtain the scattering cross section, is utterly meaningless. *There is no classical cross section when the target supports chaotic dynamics, and the call for a complementary statistical theory for ECD is rooted already in classical dynamics.*

To each scattering event in the crystals there corresponds a radiative perturbation to the aether, containing both advanced and retarded components. These ‘aether waves’ propagate inside and around the interferometer, slightly perturbing the (locally almost) classical path of the cores, in a way which *globally* depends on the configuration of the interferometer. As the dynamics of the cores inside the crystals is chaotic, the aether excitations, their small local effect notwithstanding, have a dramatic effect on the final scattering direction of the particle.

The chaoticity of the underlying classical dynamics is crucial for the operation of the interferometer. Suppose we remove BS2 from the apparatus (b). The influence of the aether excitations on the dynamics of a particle passing in region R is now negligible, and the particle continues its straight classical path, almost unperturbed, as follows from momentum conservation. This should be contrasted with (c), ‘surrealistic’ trajectories predicted by Bohmian mechanics, taking the other direction [2].

We have focused our discussion on a crystal BS as the arena for this chaotic dynamics but, in fact, it is not chaoticity itself — a classical notion — which is essential for the operation of the interferometer, but rather the sensitivity of chaotic dynamics to perturbations. All interferometers, whether electronic or atomic, use BS’s in the form of highly sensitive devices (usually involving the spin of the particle) facilitating the amplification of the small aether induced perturbations to the core’s dynamics.

3.1.4 The ensemble current

The above description of interferometers invites a troubling question. As is well known, interferometers can be tuned to produce nearly deterministic results, with one detector firing some 99% of the times and the other only 1%. If the local dynamics of the particles are so nearly classical, viz. locally defined, then how do they acquire this destiny, of arriving

predominantly at one detector rather than the other? (or exit the crystals at the Bragg angles only?)

To answer this question, we first need to see what a scattering experiment is, in the context of ECD. Let $j \in \mathcal{E}$ be the regular electric current associated with a solution realized in the experiment (we drop the ^r superscript in this section), and \mathcal{E} the ensemble of all such currents. An experiment is seen as a realization of a measure $d\mu(j)$ defined on \mathcal{E} , namely, we assume that as the number, n , of scattered particles goes to infinity, the number of solutions realized in any subset $\Sigma \in \mathcal{E}$ approaches $n\mu(\Sigma) \equiv n \int_{\Sigma} d\mu(j)$. The reader can verify that the scattering cross section as well as any other measurable statistical expression produced by single-body QM, such as the spectrum of atoms, can be read from an ordinary, conserved, four-current — the *ensemble current* ⁶,

$$j_{\text{ens}} = \int_{\mathcal{E}} d\mu(j) j. \quad (57)$$

Stated in the above terminology, then, the question of interest is why does this current have such an asymmetric form? The answer to the question does not lie in four-dimensional Minkowski's space-time, on which j_{ens} is defined, but rather in infinite-dimensional \mathcal{E} , the domain of μ . A single 'point' in \mathcal{E} — a current j — is such a complex, non locally defined object, that we lack any intuition regarding sensible distributions thereof. Why is 99% — 1% less intuitive than 50% — 50%? Likewise, why is the nonuniform shape of the Hydrogen-atom spectrum counter intuitive? Note that in both cases, no classical counter proposal even exists. In the scattering case, the standard procedure of averaging over the impact parameter leads to a meaningless result when applied to chaotic systems. As to the spectrum — a classical Hydrogen atom is a meaningless concept to begin with.

In fact, the measure μ should be regarded as an *independent law of nature*, on equal footing with ECD itself, constrained only by compatibility requirements with ECD and the experimental settings (try thinking what would constitute a natural μ ?). For this reason, QM enjoys a similar status of an independent law.

⁶The differential scattering cross-section to a given solid angle $d\Omega$ around Ω , for example, is easily deducible from the ensemble current (57). It is just

$$\frac{1}{Q d\Omega} \lim_{x^0 \rightarrow \infty} \int_C d^3 \mathbf{x} j_{\text{ens}}^0, \quad (56)$$

with $Q = \int d^3 \mathbf{x} j^0$ the conserved charge of the particle, and $C = C(d\Omega, \Omega)$ the cone in three space defined by $d\Omega$ and Ω . This follows upon inserting expression (57) into (56). In the limit $x^0 \rightarrow \infty$, every $j^0(x^0, \mathbf{x})$ is entirely supported in C , or in its complement. The \mathbf{x} integration then extracts $Q \chi_{\Sigma}(j)$ with $\Sigma = \Sigma(C) \in \mathcal{E}$ the subset of solutions scattering to cone C , and $\chi_{\Sigma}(\cdot)$ its characteristic function. The result is therefore $(d\Omega)^{-1} \int_{\mathcal{E}} d\mu(j) \chi_{\Sigma}(j) \equiv (d\Omega)^{-1} \mu(\Sigma)$ which is the definition of the differential cross-section.

As yet another example, consider the EM spectrum emitted by a heated gas. For a sufficiently dilute gas, the currents associated with the bound electrons (those generating the radiation) can safely be assumed to constitute an incoherent ensemble \mathcal{E} . By the linearity of Maxwell's equation, and the incoherence assumption, the spectrum produced by the ensemble current equals the sum of spectra produced by the individual currents in \mathcal{E} . Equivalently, \mathcal{E} can comprise different, sufficiently remote time segments, of the current associated with a single atom. The spectral peaks, then, appear simply as dominant frequencies in the dipole radiation of j_{ens} , representing *statistically* more common frequencies in the dipole radiation of members in the ensemble.

3.1.5 Relativistic wave equations

Single-particle QM, as argued above, describes very coarse aspects of the measure μ — very ‘low order moments’ of that infinite dimensional distribution. It should not come as too great a surprise that, assuming ECD is indeed the physics prevailing at the atomic scale, QM could have been anticipated independently of ECD, with the latter’s very unique content. We shall next show why relativistic wave equations, such as the first or second order Dirac equation, and the Klein-Gordon equation, are a natural tool for guessing those moments, in certain cases of a single particle moving in an external field.

Consider, then, the ECD solution of a single particle in the external field F_{ext} . Let this solution be indexed by the electric current j , associated with the particle, and let ${}^j m$ be the corresponding e-m tensor. From (47) we have

$$\partial_\nu {}^j m^{\nu\mu} = (F_{\text{ext}}^{\mu\nu} + {}^j F_{\text{sel}}^{\mu\nu}) j_\nu, \quad (58)$$

with ${}^j F_{\text{sel}}$ the self-field generated by j ,

$$\partial_\nu {}^j F_{\text{sel}}^{\nu\mu} = j^\mu. \quad (59)$$

Multiplying (58) by $d\mu(j)$ and integrating over \mathcal{E} , we first make the assumption that the contribution of the self-fields,

$$\int_{\mathcal{E}} d\mu(j) {}^j F_{\text{sel}}^{\mu\nu}(x) j_\nu(x), \quad (60)$$

can be neglected, compared with that of the external field. This is a reasonable assumption for a sufficiently incoherent ensemble, as then the self contribution of different members in the ensemble, to the self force at a point x , enters with a nearly random orientation. For this to happen, however, the different charges must not radiate (advanced or retarded fields) in preferred directions, hence the limitation of the ensemble current approach and of relativistic wave equations in particular (see more in section 3.2.1). Note, nonetheless, that the self-fields are dominant in the individual currents j and ${}^j m$, even when their contributions to the integral over the ensemble have been neglected, guaranteeing that self-force effects are not eliminated in that process. In particular, the effective mass and charge of the particles, strongly depend on that self field.

With the above approximation, we get the following four relations

$$F_{\text{ext}}^\mu j_{\text{ens}}^\nu = \partial_\nu m_{\text{ens}}^{\nu\mu}, \quad \text{with} \quad m_{\text{ens}} = \int_{\mathcal{E}} d\mu(j) {}^j m, \quad (61)$$

and a conservation constraint

$$\partial_\nu j_{\text{ens}}^\nu = 0, \quad (62)$$

inherited from the conservation of the individual j . The Lorentz vector and second rank tensor, j_{ens} and m_{ens} resp., must obviously transform like their constituents in any symmetry transformation belonging to the symmetry group of ECD.

Consider now a low energy scattering experiment. As shown above, the scattering cross section can be computed from j_{ens} . However, a similar construction applied to m_{ens} can also produce the cross section, which *must coincide* with that computed using j_{ens} . This relation adds up to (61), (62) and the symmetry group of ECD, producing a very restrictive condition on the set of permissible pairs $\{j_{\text{ens}}, m_{\text{ens}}\}$, regardless of the details of the ECD dynamics.

A systematic way of producing such constrained pairs, enjoying the full symmetry group of ECD, is via relativistic wave equations⁷. In the scalar case, the relevant equation is the Klein-Gordon equation

$$(D^2 + \hat{m}^2) \psi = 0, \quad (65)$$

with the gauge covariant derivative

$$D = \hat{h}\partial - i\hat{q}A, \quad (66)$$

where \hat{h} , \hat{q} and \hat{m} are some constants, and A the external EM potential. The expressions for the ensemble electric current

$$j_{\text{ens}}^\mu = \hat{q} \text{Im } \psi^* D^\mu \psi, \quad (67)$$

and the ensemble e-m tensor

$$m_{\text{ens}}^{\nu\mu} = g^{\nu\mu} \left(\frac{1}{2} \hat{m}^2 \psi \psi^* - \frac{1}{2} (D^\lambda \psi)^* D_\lambda \psi \right) + \frac{1}{2} (D^\nu \psi (D^\mu \psi)^* + \text{c.c.}), \quad (68)$$

satisfy all the above compatibility conditions — eq. (61) in particular. Eq. (61), when restricted to a field-free region, imposes certain relations between the parameters of (65), and the conserved electric charge and mass of the particles comprising the ensemble.

The wave-function ψ , then, labels an ‘irreducible ensembles’, μ_ψ , to which there corresponds an ‘irreducible pair’, $\{j_{\text{ens}}, m_{\text{ens}}\}_\psi$. A generic experiment, however, involves a few irreducible ensembles, which are sampled with different weights. This is the meaning of a ‘statistical mixture’ of wave-functions in QM. The collapse postulate of measurement theory, merely represents a transition from one ensemble to another. For example, to a beam of particles escaping a hot oven there correspond one ensemble. When the beam is further split into two by a (e.g. Stern-Gerlach) polarizer, each part must obviously be represented

⁷The more general way is via the conservation laws associated with solutions, ψ , of the five dimensional Schrödinger equation (15). Its unitarity implies

$$\partial_s |\psi|^2 = \partial \cdot J \equiv \partial \cdot (\text{Im } \psi^* D \psi), \quad (63)$$

while the Ehrenfest relations give

$$\begin{aligned} \partial_s J^\mu &= F^{\mu\nu} J_\nu - \partial_\nu M^{\nu\mu} \\ &\equiv F^{\mu\nu} J_\nu - \partial^\nu \left[g^{\nu\mu} \left(\frac{i\hbar}{2} (\psi^* \partial_s \psi - \partial_s \psi^* \psi) - \frac{1}{2} (D^\lambda \psi)^* D_\lambda \psi \right) + \frac{1}{2} (D^\nu \psi (D^\mu \psi)^* + \text{c.c.}) \right] \end{aligned} \quad (64)$$

Integrating (63) and (64) from $s = -\infty$ to $s = \infty$, we get two candidates, $j_{\text{ens}} = \int ds J$ and $m_{\text{ens}} = \int ds M$, satisfying all our requirements. These, however, correspond to ensembles with a continuum of masses, and are therefore more difficult to relate to actual experiments, involving a single particle species.

by a different ensemble, when scattered off of a spin sensitive target. If, however, the two parts of the original beam are recombined in an interferometer, the original ensemble is the relevant one.

A major historical difficulty associated with the KG field is also resolved in this framework. The non-positivity of j_{ens}^0 (motivating the Dirac equation) simply reflects the non-positivity of the individual j^0 comprising j_{ens} . It is only the space integral over those individual components, representing the total charge, that is guaranteed to remain constant.

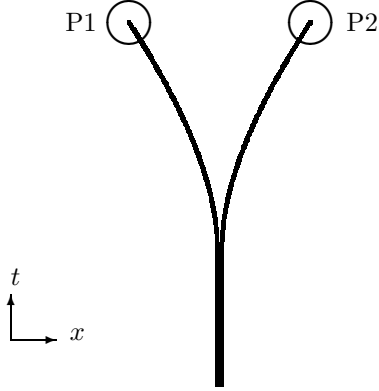
3.2 Many-body ECD

In the previous section, the EM potential was divided into an external potential, generated by all particles but one, plus a self potential, due entirely to this one, privileged, particle. This division is legitimate on the premise that the self potential of the privileged particle does not alter the solutions of the rest of the particles, which is not the case when the privileged particle interacts with the rest of the particles, either ‘electrostatically’, viz. at close range, or ‘radiatively’, via long-range aether waves.

Let us begin with the first case. As explained in section 2.2.3, the self consistent potential entangles closely interacting particles in such a way that one can no longer regard matter as a composition of individual particles but, instead, as some ‘self consistent matter-radiation condensate’. What may seem surprising at first is that long after their separation, and at arbitrarily remote locations, two particles which have closely interacted in the past, ‘bear the memory’ of their encounter.⁸

Consider, for example, two nucleons, escaping a nucleus, arriving each at a polarimeter (P1 and P2).

⁸As entanglement is transitive, all the particles in a dense cloud are entangled, and should be solved as a single space time structure. If a large scale astronomical object, such as a galaxy, passes through an epoch of a dense ‘fireball’ (not necessarily anything as dramatic as a big bang or big crunch — whatever that means) it could reasonably be that its morphology and dynamics at much latter stages, reflect that entangled epoch. As it is also reasonable to assume that galaxies indeed pass through some fireball epoch, gravitation on galactic scales (and possibly above) could therefore be due to such a primordial entanglement, and need not even resemble solar scale gravity (the largest scale in which general relativity has been directly confirmed). This would have obvious implications on the current interpretation of astronomical data. In this regard, we should also notice another possibility opened by ECD — scale drift. As shown in appendix A, both the mass of individual ECD particles, as well as the scale charge of their combined solution, may slowly drift over time. This offers an alternative explanation for the source of galactic redshifts. In fact, a universe collectively increasing its scale leads to a Hubble-like relation, as light collected from remote galaxies is emitted at an epoch of lower mass (hence longer wavelength) which is proportional to the distance of the emitter to the observer, for *all* observers.



If the two polarimeters are positioned sufficiently far apart, then the ECD system, self potential included, can be solved independently for each particle. This is a consequence of $\lim_{s \rightarrow \infty} G(x, x', s) = 0$, suppressing the large $|s - s'|$ contribution to the s' -integral in (13). There is therefore nothing unique about the dynamics of each particle, giving away their histories, which is washed away over macroscopic scales. The above independence notwithstanding, if one tries to continue those independent solutions into the past, then at some point, when the two particles come close, it could become impossible to combine the two solutions into a single, self consistent one, unless each of the two independent solutions is restricted to a certain subset of the full set of independent solutions — a subset which obviously depends on the orientation of *both* polarimeters. Given the orientations of the two polarimeters, therefore, the combined solution of the two particles in the above example, must be solved as a whole — as a single space-time structure.

To each orientation choice, p_1, p_2 , for P1 and P2 resp., there corresponds a *different* ensemble, \mathcal{E} , of two-particle ECD solutions, equipped with its own measure, μ . This contradicts Bell's assumption in deriving his celebrated inequalities, which maintains that the *same* ensemble of hidden variables be used, irrespective of the orientations of the polarimeters (roughly corresponding to the fact that the particles should not 'anticipate' the orientations of the polarimeters before encountering them).⁹

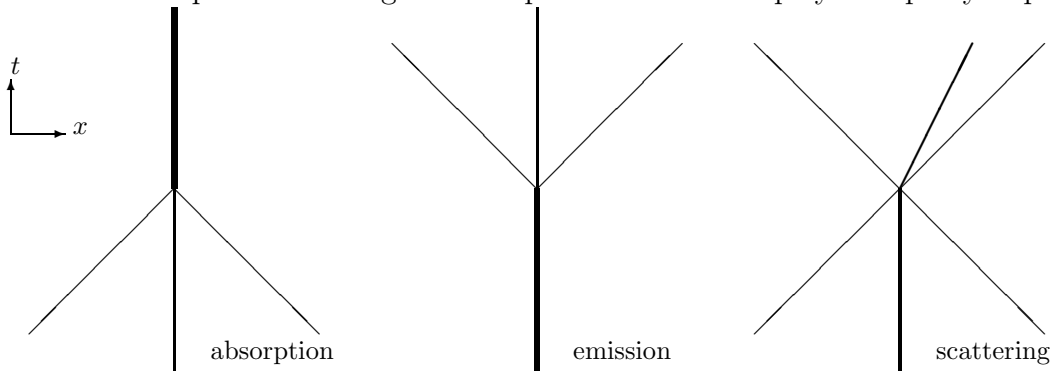
Like its single-body counterpart, the measure, μ , enjoys the status of an independent law of nature, on equal footing with ECD itself. However, a simple generalization of the ensemble current to the case of many-body ECD, probably doesn't exist, hence the enormous complication of many-body relativistic QM — quantum field theory — involving both matter and the EM potential.

Finally, let us note that the same discussion holds also for two particles, initially separated, which later bind together. This scenario, however, does not correspond to known experiments.

⁹The above mechanism, accounting for violations of Bell's inequalities, is in the spirit of so called 'retro-causal' models. See, e.g. [1] and extensive references therein.

3.2.1 The conspiracy of the photon

Perhaps the strongest motivation for the introduction of photons, is the salvation of energy-momentum conservation. Indeed, the photoelectric and Compton's effects are manifestly in violation of classical energy-momentum conservation. More specifically, equation (52), expressing the change in the four-momentum of a particle as a function of the integrated e-m flux across a space-like surface surrounding the particle, can formally be applied to the corresponding classical currents as well. In the case of the photoelectric or Compton's effects, the e-m flux across T , identical with the Poynting vector, is computed from the external trigger, F_{ext} , and a possible retarded outgoing wave, generated in the jolting of the charge. As both effects are observed even for extremely feeble triggers, the contribution of F_{ext} to the Poynting vector may be neglected, while that of the retarded wave can be shown to be positive. We may then get an arbitrarily large energy deficit at time following the jolting of the charge. In ECD, on the other hand, we saw that advanced e-m waves must be included in the analysis. Thus, for example, 'photon absorption' by a molecule, should correspond to predominantly incoming advanced e-m waves, converging on the molecule and increasing its internal energy (or ionizing it as in the photoelectric effect). In 'spontaneous emission', it is rather outgoing retarded waves removing energy from the molecule. In other situations — Compton scattering for example — both waves play an equally important role.



The above two features, viz. automatic selection of the correct radiation field (guaranteeing energy-momentum conservation), along with its incorporation into the dynamics of the particle (no self-force problem), are missing from classical electrodynamics, hence the need for ECD to implement this, otherwise, classical idea.

Being highly nonlocal and nonlinear, there are plenty of pairs $\{\phi, \gamma\}$, solving the ECD system (self potential included) for a given external trigger. The distribution of the corresponding currents can be read from the appropriate ensemble-current (see section 3.1.4) which, in the case of the photoelectric effect, gives the well known¹⁰ result that the electron

¹⁰This calculation is usually preformed with the non-relativistic Schrödinger equation, considering the incident wave as a small perturbation. However, for wavelength much smaller than the electron's Compton length, the Dirac equation gives identical results.

The success of the ensemble current formalism in the case of the photoelectric effect, is due to the isotropic distribution of the direction of the ejected particle, hence also of the corresponding self force, justifying the omission of the term (60). In contrast, this formalism fails when applied to Compton scattering, i.e. an external EM trigger in the form a plane wave, but without a heavy trap holding the particle. Momentum

either jolts with energy, $\lambda\omega$, proportional to the frequency, ω , of the incident radiation (ignoring for simplicity the binding energy), or else does not jolt at all. This binary response of electrons, typical of *all* ‘*photodetectors*’ *by definition* (or else they are called calorimeters, antennas, etc.), is the historical reason for the introduction of photons. It is as if a ‘light corpuscle’ of energy $\lambda\omega$ has struck the jolted electron. Yet another standard result emerging from the analysis of the ensemble current, is that the probability for a jolting event is proportional to the amplitude squared of the incident wave, implying that the probability drops as the inverse of the distance squared between source and detector — just as if a flux of particles is erupting from the emitter.

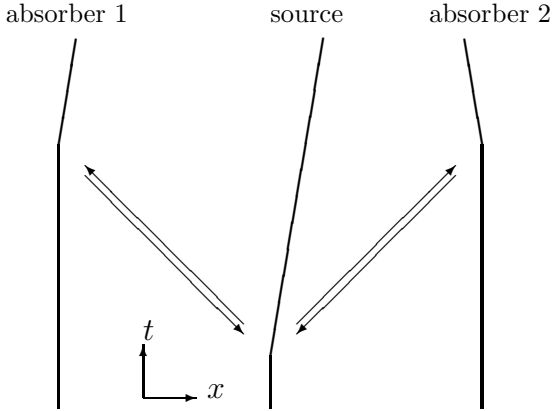
But the analogy with other particles goes even further. A typical example involves a so called ‘single-photon source’ or more generally an n -photon source (Fock state source), e.g. a molecule excited by a femtoseconds laser pulse, and then allowed to spontaneously decay. If the source is surrounded by a large sphere, consisting of independently operating photodetectors (which can further be prevented from cross-talking by, e.g. partitions) then the above results of the ensemble current, imply that the *average* number of photodetections does not depend on the radius of the sphere, and is entirely an attribute of the source (note that, as the expectation value is additive even for dependent random variables, this result is not altered when, latter, we argue that the photodetectors are not independently operating). This, again, is consistent with a scenario of a release of a fixed number of particles in each decay of the molecule. However, the independence of the different photodetectors also imply that the number of detected photons should fluctuate around its mean with a standard deviation proportional to the square-root of the mean. For a large mean, this fluctuation may be ignored, but for a small one, it is significantly greater than the observed value which is more consistent with a fixed number of particles scenario, having no fluctuations.

As implied above, the loop-hole in the analysis is in the assumption of independence of the photodetections. While it is possible to prevent different photodetectors from cross-talking, it is, by definition, impossible to prevent each of them from cross talking with the source if advanced waves are present in the radiation fields of the absorbing charges¹¹. In actual experiments, e.g. [6], the retarded field of the source is relayed to the detecting charges by other charges, comprising mirrors, beam-splitters, fiber-optics etc. The crucial point is that, whatever optical path exists between the source and the detector, by means of retarded fields, there must necessarily exist a reverse path leading from the detector to the source via advanced fields. The source therefore serves as a hub for indirect cross-talking between the absorbing charges, leading to statistical dependence in their responses. As to why the actual fluctuation around the mean is much smaller, rather than larger, than that expected on the premise of independence — this is a statistical effect, not to be sought in ECD alone. This

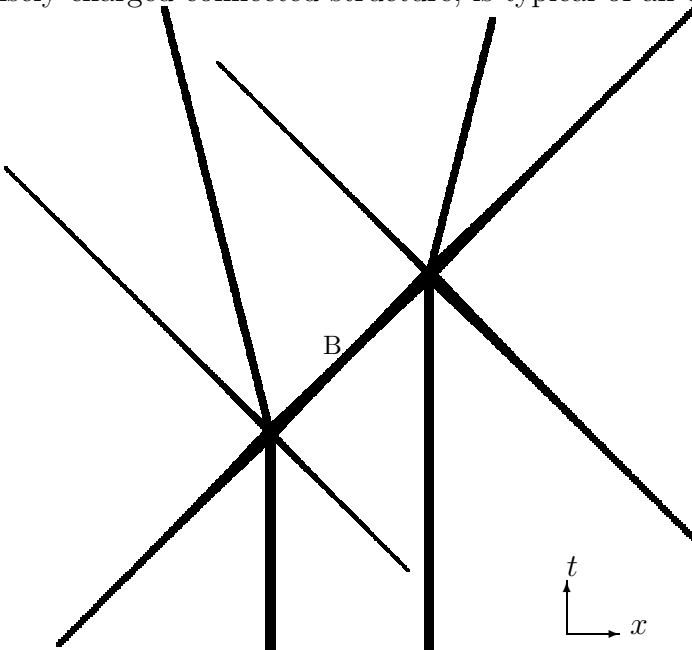
conservation — ignored in the photoelectric effect due to the large mass of the trap — dictates that the direction of the ejected charges, must be strongly correlated with that of the incident wave, and (60) cannot be neglected.

¹¹The use of advanced solutions in order to explain the non classical statistics of photons, latter receiving the name ‘the transactional interpretation of QM’, is described in [3]. Using point charges, however, that proposal does not explicitly deal with energy-momentum balance, nor with the mechanism causing a charge to jolt.

is the realm of QM — QED to be specific. Violations of Bell’s inequalities in photons pair measurements etc., are presumably all manifestations of that indirect cross-talking.



We see how various features of ECD and QM may have conspired to bring about the illusion that ‘light particles’ must be involved in radiation processes (one can potentially extend the above arguments to other neutral particles, such as the neutrino). The real moral, however, lies in the geometry of Minkowski’s space and in the unity of the e-m field p . We have previously argued that the self consistency loop entangles, in the statistical sense, two particles whose associated world-cylinders, supporting the lions share of their charges, have a significant overlap in \mathbb{M} . This can be seen as a manifestation of the fact that, fundamentally, the value of p at a point cannot be attributed to any single particle, not even inside the dense cylinders associated with the particle. The conclusion deduced from that example generalizes to the observation that any connected volume in \mathbb{M} , of a sufficiently high e-m density must be treated as a single space-time structure. In particular, the following densely charged connected structure, is typical of all emitter-absorber ‘transactions’.



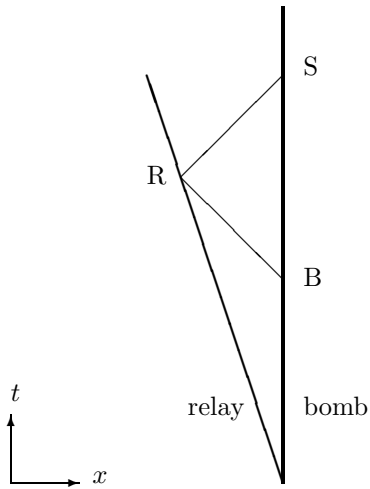
The term ‘transaction’ is deliberately borrowed from [3] as it highlights the symmetric role

played by both charges appearing in the structure, viz. the absorber may just as well be seen as the cause, triggering the emission via advanced waves, rather than the effect, triggered by the retarded waves of the emitter. Note that in ECD this blurring between cause and effect goes even further than in [3], as p , in particular on the ‘bridge’, B, between the two particles, cannot be decomposed into advanced plus retarded contributions, and is therefore a genuine attribute of the structure as a whole.

3.2.2 Advanced waves

The central role played by advanced waves in explaining the illusion of a photon, calls for a closer look at these disputable objects. There is a strong, largely unjustified, bias against the inclusion of advanced waves — advanced solutions of Maxwell’s equations in particular — into the description of physical reality. The main objection draws parallels with ‘contrived’ advanced solutions of other physical wave equations (e.g. surface waves in a pond converging on a point and ejecting a pebble.). This parallelism, however, is a blatant repetition of the historical mistake, which led to the invention of the aether (the historical aether, not to be confused with that used in this paper). The formal mathematical similarity between the d’Alembertian (the only Lorentz invariant second-order differential operator) and other (suitably scaled) wave operators, is no more than a mis-fortunate coincident (has this coincidence had some real substance to it, then application of the Lorentz transformation to the wave equation describing the propagation of sound, for example, would have yielded a meaningful result).

Yet another argument against advanced solutions is their alleged involvement in causal paradoxes.



Indeed, if advanced waves could be generated just like their retarded counterparts, then the following paradoxical situation could occur. A device consisting of a bomb, a transmitter, a receiver and a timer, is set to send a retarded signal at S. The signal is relayed at R, received at B, and triggers the fuse of the bomb. But if the bomb goes off at B, then no signal is sent

at S. Why then did the bomb explode? On the other hand, if the bomb does not go off at B, then a signal must be sent at S, detonating the bomb at B. Either way get a contradiction.

The resolution of the paradox should not be sought in ECD proper. Indeed, if ECD is a valid theory, then the CPT image of a radio transmitter sending retarded waves, is a radio transmitter made of antimatter sending advanced waves. A radio transmitter, however, cannot be seen as an autonomous entity. Its generated waves are eventually absorbed by other particles and, as argued above, the emission of waves cannot be separated from their absorption (as in the Wheeler-Feynman absorber theory, [7]). The privileged status of retarded waves in all *macroscopic* radiation processes, is therefore an attribute of the specific *solution* of the ECD equations (selected, among else, by the anthropic principal), representing the local part of the universe we live in, and is intimately connected with the excess of matter over antimatter around us.

It seems, then, that the strongest case against advanced solutions is observational. While spontaneous emission or absorption may be seen as direct evidences to the contrary, if advanced solutions played a dominant role in *any* photo-absorption (as implied e.g. in the figure on page 27) then their prevalence should have matched that of their retarded counterparts, leaving a striking signature on all radiation processes. Let us then see why this needs not be the case. Recall that our primary motivation for introducing advanced solutions was to salvage energy-momentum conservation. In the example from the previous section of a source surrounded by photodetectors, the integrated flux of energy falling on any single photodetector, must be smaller than the EM energy released by the source. If that energy equals $\lambda\omega$, advanced waves must be invoked in order to account for the firing of a photodetector, as this amounts to increasing the energy of an electron by $\lambda\omega$. However, if the above ‘single photon source’ is replaced by a continuous light source of arbitrary intensity, then the following process, not involving advanced waves, can be envisaged. Retarded waves, originating from the source, arrive at the photodetector, which slowly absorbs them. In the process of absorption the electrons in the device radiate in the same direction as that of the incident radiation, but in a phase which interferes destructively with the latter, slowly ‘removing’ energy from the incident wave. This is essentially the classical description of radiation absorption, only in ECD, the energy extracted from the incident wave needs not appear instantly as kinetic energy. The extended support of the ECD energy density, in conjunction with its dynamical evolution, support a scenario in which energy is gradually accumulated by the charge in the form of latent ‘internal’ energy, and is rapidly converted into kinetic energy only when a threshold, equal to $\lambda\omega$, has been crossed. That the conversion of latent energy into kinetic energy happens at the $\lambda\omega$ threshold can, again, be read from the ensemble current which, as remarked before, is indifferent to the mechanism shooting the individual electrons.

Remarkably, it is known that the statistics of photodetection also changes when shifting to a continuous source. When the readings of two photodetectors are correlated (as in [6]), the anticorrelation consistent with a particle scenario, turns into the expected positive correlation when the single photon source is replaced by a continuous light source of thermal origin, or to (the equally intuitive) vanishing correlation when strongly attenuated laser light

is used. It appears, therefore, that advanced waves play a dominant role only in sufficiently ‘delicate’ radiation process, involving energy transfer on the order of $\lambda\omega$. Such processes are overwhelmed by ordinary radiation processes involving a huge number of particles, such as the burning of a candle, or in lasing devices.

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Appendices

A Conservation of ECD currents

To prove the conservation of the regular current, j^r , defined in (33), we first need the following lemma, whose proof is obtained by direct computation.

Lemma. Let $f(x, s)$ and $g(x, s)$ be any (not necessarily square integrable) two solutions of the homogeneous Schrödinger equation (15), then

$$\frac{\partial}{\partial s}(fg^*) = \partial_\mu \left[\frac{i}{2}(D^\mu fg^* - (D^\mu g)^* f) \right]. \quad (69)$$

This lemma is just a differential manifestation of unitarity of the Schrödinger evolution—hence the divergence.

Turning now to equation (13), written for the rescaled wave-function $\epsilon^{-1}\phi$

$$\phi(x, s) = -2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds' G(x, \gamma_{s'}; s - s') f^R(s') \mathcal{U}(\epsilon; s - s'), \quad (70)$$

and its complex conjugate,

$$\phi^*(x, s) = 2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds'' G^*(x, \gamma_{s''}; s - s'') f^{R*}(s'') \mathcal{U}(\epsilon; s - s''), \quad (71)$$

we get by direct differentiation

$$\begin{aligned} q \frac{\partial}{\partial s} & \left[-2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \right. \\ & \quad \left. \mathcal{U}(\epsilon; s - s') G(x, \gamma_{s'}; s - s') \mathcal{U}(\epsilon; s - s'') G^*(x, \gamma_{s''}; s - s'') \right] \\ & = -2q\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \\ & \quad \partial_s [G(x, \gamma_{s'}; s - s') G^*(x, \gamma_{s''}; s - s'')] \mathcal{U}(\epsilon; s - s') \mathcal{U}(\epsilon; s - s'') \\ & + [\partial_s \mathcal{U}(\epsilon; s - s') \mathcal{U}(\epsilon; s - s'') + \mathcal{U}(\epsilon; s - s') \partial_s \mathcal{U}(\epsilon; s - s'')] \\ & \quad G(x, \gamma_{s'}; s - s') G^*(x, \gamma_{s''}; s - s''). \end{aligned} \quad (72)$$

Focusing on the first term above, we note that, as G is a homogeneous solution of Schrödinger's equation, we can apply our lemma to that term, which therefore reads

$$\begin{aligned}
& -2q\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \\
& \partial_\mu \left[\frac{i}{2} (D^\mu G(x, \gamma_{s'}; s-s') G^*(x, \gamma_{s''}; s-s'') - (D^\mu G(x, \gamma_{s''}; s-s''))^* G(x, \gamma_{s'}; s-s')) \right] \\
& \mathcal{U}(\epsilon; s-s') \mathcal{U}(\epsilon; s-s'').
\end{aligned} \tag{73}$$

Integrating (72) with respect to s , the left-hand side vanishes (we can safely assume it goes to zero for all x, s', s'' as $|s| \rightarrow \infty$), and the derivative ∂_μ can be pulled out of the triple integral in the first term. The reader can verify that this triple integral is just $\partial_\mu j^\mu$, with j given by (11) and ϕ, ϕ^* are explicated using (70), (71). The regular current, (33), is therefore conserved, provided the s integral over the second term in (72) is missing an ϵ^0 term in its ϵ -expansion.

Let us then show that, in the distributional sense, this is indeed the case. Integrating the second term with respect to s , and using $\partial_s \mathcal{U}(\epsilon; s-s') = \delta(s-s'-\epsilon) + \delta(s-s'+\epsilon)$, that term reads

$$\begin{aligned}
& -2q\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds' f^R(s') \quad 2\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds'' f^{R*}(s'') \\
& \mathcal{U}(\epsilon; s'-\epsilon-s'') G(x, \gamma_{s'}; -\epsilon) G^*(x, \gamma_{s''}; s'-\epsilon-s'') \\
& + \mathcal{U}(\epsilon; s'+\epsilon-s'') G(x, \gamma_{s'}; +\epsilon) G^*(x, \gamma_{s''}; s'+\epsilon-s'') \\
& + \mathcal{U}(\epsilon; s''-\epsilon-s') G(x, \gamma_{s'}; s''-\epsilon-s') G^*(x, \gamma_{s''}; -\epsilon) \\
& + \mathcal{U}(\epsilon; s''+\epsilon-s') G(x, \gamma_{s'}; s''+\epsilon-s') G^*(x, \gamma_{s''}; +\epsilon).
\end{aligned} \tag{74}$$

Using (70) and (71), this becomes

$$\text{Re} \quad -4q\pi^2\bar{h}^2i \int_{-\infty}^{\infty} ds' f^R(s') \left[\phi^*(x, s'-\epsilon) G(x, \gamma_{s'}; -\epsilon) + \phi^*(x, s'+\epsilon) G(x, \gamma_{s'}; \epsilon) \right]. \tag{75}$$

Writing $\phi = \epsilon^{-1}\phi^s + \phi^r$ above, and using the short- s propagator (19) plus the explicit form, (22), of ϕ^s , one can obtain the ϵ -expansion of (75). Expanding first $\phi^{s*}(x, s \pm \epsilon)$ in powers of ϵ , the part of the integrand involving ϕ^s can be shown to comprise an ϵ -independent term multiplying $\epsilon^{-2}f_s(\xi^2/2\bar{h}\epsilon)$, with $f_s(y) = \text{sinc}(y) \cos(y) = f_s(-y)$, and another ϵ -independent term multiplying $\epsilon^{-3}f_a(\xi^2/2\bar{h}\epsilon)$, with $f_a(y) = \text{sinc}(y) \sin(y) = -f_a(-y)$. Using the evenness and oddness of f_s and f_a resp., the first term behaves for small ϵ like $\epsilon^{-1}\delta(\xi^2) + O(\epsilon)$, while the second — as $\epsilon^{-1}\delta'(\xi^2) + O(\epsilon)$, both, therefore, do not involve the ϵ^0 coefficient, which is due entirely to ϕ^r . Using (16), the latter's contribution reads in the limit $\epsilon \rightarrow 0$

$$\begin{aligned}
& -8q\pi^2\bar{h}^2 \int_{-\infty}^{\infty} ds \text{Re} \quad i f^R(s) \phi^{r*}(\gamma_s, s) \delta^{(4)}(x - \gamma_s) = \\
& 8q\pi^2\bar{h}^2 \int_{-\infty}^{\infty} ds \text{Im} \quad f^R(s) \phi^{r*}(\gamma_s, s) \delta^{(4)}(x - \gamma_s).
\end{aligned} \tag{76}$$

This is a distribution, supported on $\bar{\gamma}$, which vanishes by virtue of (27). We have therefore shown that $\partial \cdot j^r = 0$ *in the distributional sense*. This is enough to establish the time-independence of the charge, as one only needs to integrate $\partial \cdot j^r = 0$ over a volume in Minkowski's space, and apply Stoke's theorem, to get a conserved quantity. But, in fact, it is easily shown that j^r is a smooth function in the limit $\epsilon \rightarrow 0$, implying a pointwise identity $\partial \cdot j^r = 0$.

To gain a more explicit geometrical insight into the meaning of a 'line sink in Minkowski's space', consider a small space-like three-tube, T , surrounding $\bar{\gamma}$, the construction of which proceeds as follows. Let $\beta(\tau) = \gamma(s(\tau))$ be the world line $\bar{\gamma}$, parametrized by proper time $\tau = \int^s \sqrt{(d\gamma)^2}$, and let $x \mapsto \tau_r$ be the *retarded light-cone map* defined by the relations

$$\eta^2 \equiv (x - \beta_{\tau_r})^2 = 0, \quad \text{and} \quad \eta^0 > 0. \quad (77)$$

Let the 'retarded radius' of x be

$$r = \eta \cdot \dot{\beta}_{\tau_r}. \quad (78)$$

Taking the derivative of (77), treating τ_r as an implicit function of x , and solving for $\partial \tau_r$, we get

$$\partial \tau_r = \frac{\eta}{r} \Rightarrow \partial r = \dot{\beta}_{\tau_r} - \left(1 + \ddot{\beta}_{\tau_r} \cdot \eta\right) \frac{\eta}{r}. \quad (79)$$

The (retarded) three-tube of radius ρ is defined as the space-like three surface

$$T_\rho = \{x \in \mathbb{M} : r(x) = \rho\}.$$

It can be shown in a standard way that the directed surface element normal to $x \in T_\rho$ is

$$d^\mu T_\rho = \partial^\mu r|_{r=\rho} \rho^2 d\tau d\Omega, \quad (80)$$

where $d\Omega$ is the surface element on the two-sphere.

Let Σ_1 and Σ_2 be two time-like surfaces, intersecting T_ρ and T_R . Applying Stoke's theorem to the interior of the three surface composed of T_ρ , T_R , Σ_1 and Σ_2 , and using $\partial \cdot j^r = 0$ there, we get

$$\int_{\Sigma_2} d\Sigma_2 \cdot j^r + \int_{\Sigma_1} d\Sigma_1 \cdot j^r = - \int_{T_\rho} dT_\rho \cdot j^r - \int_{T_R} dT_R \cdot j^r. \quad (81)$$

Realistically assuming that the second term on the r.h.s. of (81) vanishes for $R \rightarrow \infty$, we get that the 'leakage' of the charge, $\int_{\Sigma_2} d\Sigma_2 \cdot j^r - \int_{\Sigma_1} d\Sigma_1 \cdot j^r$, equals to $-\lim_{\rho \rightarrow 0} \int_{T_\rho} dT_\rho \cdot j^r$.

As $dT_\rho = O(\rho^2)$, the leakage only involves the piece of j^r diverging as r^{-2} . This piece, reads

$$\begin{aligned} & 2q\bar{h}^2 \int ds \operatorname{Im} \phi^{r*}(x, s) f^R(s) \partial \frac{1}{2\bar{h}\epsilon} \operatorname{sinc} \left(\frac{\xi^2}{2\bar{h}\epsilon} \right) \xrightarrow{\epsilon \rightarrow 0} 2q\bar{h}^2 \pi \int ds \operatorname{Im} \phi^{r*}(x, s) f^R(s) \partial \delta(\xi^2) \\ & \sim 2q\bar{h}^2 \pi \partial \int ds \operatorname{Im} \phi^{r*}(\gamma_s, s) f^R(s) \delta(\xi^2) = q\bar{h}^2 \pi \sum_{s=s_r, s_a} \operatorname{Im} \phi^{r*}(\gamma_s, s) f^R(s) \partial \frac{1}{|\xi \cdot \dot{\gamma}_s|}, \end{aligned}$$

where $s_r = s(\tau_r)$, and γ_{s_a} is the corresponding *advanced* point on $\bar{\gamma}$, defined by

$$\xi^2 \equiv (x - \gamma_{s_a})^2 = 0, \quad \xi^0 < 0.$$

Focusing first on the contribution of s_r , and using a technique similar to that leading to (79), we get

$$\partial \frac{1}{\xi \cdot \dot{\gamma}_{s_r}} = -\frac{\dot{\gamma}_{s_r}}{(\xi \cdot \dot{\gamma}_{s_r})^2} + \frac{(\dot{\gamma}_{s_r}^2 + \ddot{\gamma}_{s_r} \cdot \xi) \xi}{(\xi \cdot \dot{\gamma}_{s_r})^3} \underset{\xi \rightarrow 0}{\sim} -\frac{\dot{\beta}_{\tau_r}}{mr^2} + \frac{\eta}{mr^3}, \quad (82)$$

where $m = d\tau/ds$ needs not be constant. In the limit $\rho \rightarrow 0$, using $\partial \frac{1}{\xi \cdot \dot{\gamma}_{s_r}} \cdot \partial r|_{r=\rho} \rightarrow m^{-1}$, the contribution of s_r to the flux across T_ρ is most easily computed

$$\begin{aligned} \int_{T_\rho} dT_\rho \cdot j^r &= q\bar{h}^2\pi \int d\Omega \int d\tau_r m^{-1} \text{Im } \phi^{r*}(\beta_{\tau_r}, \tau_r) f^R(\tau_r) \\ &= 4q\bar{h}^2\pi^2 \int ds_r \text{Im } \phi^{r*}(\beta_{s_r}, s_r) f^R(s_r). \end{aligned} \quad (83)$$

The contribution of s_a to the flux of j^r is more easily computed across a *different*, (advanced) T_ρ , and gives the same result in the limit $\rho \rightarrow 0$. The fact that ρ can be taken arbitrarily small, in conjunction with the conservation of $j^r(x)$ for $x \notin \bar{\gamma}$, implies that the flux of j^r across *any* three-tube, $T = \partial C$, with C a three-cylinder containing $\bar{\gamma}$, equals twice the value in (83), when C is shrunk to $\bar{\gamma}$. Changing the dummy variable $s_r \mapsto s$ in (83), the formal content of (76) receives a clear meaning using Stoke's theorem

$$\int_C d^4x \partial \cdot j^r = 8q\bar{h}^2\pi^2 \int ds \text{Im } \phi^{r*}(\beta_s, s) f^R(s) \int_C d^4x \delta^{(4)}(x - \gamma_s) = \int_T dT \cdot j^r,$$

which vanishes by virtue of (27).

A.1 Energy-momentum conservation

The conservation of the ECD energy momentum tensor can be established by the same technique used in the previous section. To explore yet another technique, as well as to illustrate the role played by symmetries of ECD in the context of conservation laws, consider the following functional

$$\mathcal{A}[\varphi] = \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \frac{i\bar{h}}{2} (\varphi^* \partial_s \varphi - \partial_s \varphi^* \varphi) - \frac{1}{2} (D^\lambda \varphi)^* D_\lambda \varphi, \quad (84)$$

and let $\phi(x, s)$ be given by (70) for some fixed $A(x)$ and γ_s . Using

$$(i\partial_s - \mathcal{H})\phi = 2\pi^2\bar{h}^2 \left[G(x, \gamma_{s-\epsilon}; +\epsilon) f^R(s-\epsilon) + G(x, \gamma_{s+\epsilon}; -\epsilon) f^R(s+\epsilon) \right], \quad (85)$$

directly following from the definition of ϕ , we calculate $\mathcal{A}[\phi + \delta\phi]$ and, after some integrations by parts, we get for the first variation

$$\delta\mathcal{A} = \text{Re} \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x 4\pi^2\bar{h}^2 \left[G(x, \gamma_{s-\epsilon}; +\epsilon) f^R(s-\epsilon) + G(x, \gamma_{s+\epsilon}; -\epsilon) f^R(s+\epsilon) \right] \delta\phi. \quad (86)$$

Choosing $\delta\phi = \partial\phi \cdot a$, corresponding to $\phi(x, s) \mapsto \phi(x + a, s)$, with infinitesimal $a(x, s)$, vanishing sufficiently fast for large $|x|$ and $|s|$, so as to render $\delta\mathcal{A}$ well defined, we get in a standard way

$$\delta\mathcal{A} = \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x (\partial_\nu m^{\nu\mu} - F^\mu{}_\nu j^\nu) a_\mu \stackrel{\text{by eq. (86)}}{=} \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \operatorname{Re} 4\pi^2 \bar{h}^2 [G(x, \gamma_{s-\epsilon}; +\epsilon) f^R(s-\epsilon) + G(x, \gamma_{s+\epsilon}; -\epsilon) f^R(s+\epsilon)] \partial^\mu \phi^*(x, s) a_\mu, \quad (87)$$

with j given by (11) and

$$m^{\nu\mu} = \int_{-\infty}^{\infty} g^{\nu\mu} \left(\frac{i\hbar}{2} (\phi^* \partial_s \phi - \partial_s \phi^* \phi) - \frac{1}{2} (D^\lambda \phi)^* D_\lambda \phi \right) + \frac{1}{2} (D^\nu \phi (D^\mu \phi)^* + \text{c.c.}) ds. \quad (88)$$

The integrand above, as also the integrands appearing in the definitions of all other ECD currents, can be shown to be a distribution which becomes increasingly focused on the light-cone of γ_s for increasing distance from γ_s . Writing $\phi = \epsilon^{-1} \phi^s + \phi^r$ in (87), and using the short- s propagator (19) plus the explicit form, (22), of ϕ^s , one can obtain the ϵ -expansion of (87). The regular part of the second line (viz., coefficient of ϵ^0) only involves ϕ^r . In the limit $\epsilon \rightarrow 0$ it reads

$$\begin{aligned} & 8\pi^2 \bar{h}^2 \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \operatorname{Re} f^R(s) \delta^{(4)}(x - \gamma_s) \partial \phi^{r*}(\gamma_s, s) \cdot a(x, s) = \\ & 8\pi^2 \bar{h}^2 \int_{-\infty}^{\infty} ds \operatorname{Re} f^R(s) \partial \phi^{r*}(\gamma_s, s) \cdot a(\gamma_s, s) \end{aligned} \quad (89)$$

which vanishes by virtue of (26) *for any* a . The arbitrariness of a implies that the regular part of the expression in brackets, in the first line of (87), vanishes in the distributional sense,

$$\partial_\nu m^{\nu\mu} - F^\mu{}_\nu j^\nu = 0, \quad (90)$$

with the regular ‘matter e-m tensor’, m^r , defined by the same procedure as j^r , viz. the coefficient of ϵ^0 in its ϵ -expansion. Just like the electric current j^r , the matter e-m tensor can easily be shown to be a smooth function of x , implying pointwise equality in (90). Equation (26), by which (89) vanishes, appears therefore as the condition that no mechanical energy or momentum leak into a sink on $\bar{\gamma}$.

Not surprisingly, m^r is not conserved, due to broken translation covariance induced by $A(x)$. To compensate for this, using Noether’s theorem, we construct an ‘equally non conserved’ radiation e-m tensor, and subtract the two. Consider, then, the following functional of $A(x)$, for fixed ${}^k j^r$, (k labels the different particles)

$$\mathcal{S}[A] = \int_{\mathbb{M}} d^4x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_k {}^k j^r \cdot A. \quad (91)$$

By the Euler Lagrange equations, we get Maxwell’s equations, (4), with $\sum_k {}^k j^r$ as a source. As before, infinitesimally shifting the argument of an extremal A , viz. $A(x) \mapsto A(x + a) \Rightarrow$

$\delta A^\mu = \partial_\nu A^\mu a^\nu$, and following a standard symmetrization procedure of the resultant e-m tensor (adding a conserved chargeless piece $\partial_\lambda (F^{\nu\lambda} A_\mu)$) leads to

$$\partial_\nu \Theta^{\nu\mu} + F^\mu{}_\nu \sum_k k j^{\nu}{}^\mu = 0, \quad (92)$$

$$\text{with} \quad \Theta^{\nu\mu} = \frac{1}{4} g^{\nu\mu} F^2 + F^{\nu\rho} F_\rho{}^\mu \quad (93)$$

the canonical (viz. symmetric and traceless) ‘radiation e-m tensor’. Summing (90) over the different particles, k , and adding to (92), we get a conserved, symmetric e-m tensor, $\partial_\nu p^{\nu\mu} = 0$, with

$$p = \Theta + \sum_k k m^{\text{r}}. \quad (94)$$

A.2 Charges leaking into world-line sinks

Both methods used above, can be applied to prove the conservation of the regular part of the mass-squared current — the counterpart of (2)

$$b(x) = \int ds B(x, s) \equiv \int ds \text{Re } \bar{h} \partial_s \phi^* D \phi, \quad \text{for } x \notin \bar{\gamma}. \quad (95)$$

In the first method, used to establish the conservation of j^{r} , the counterpart of (69) is $\partial_s (g^* \mathcal{H} f) = \partial \cdot (\text{Re } \bar{h} \partial_s g^* D f)$, corresponding to the invariance of the Hamiltonian (in the Heisenberg picture) under the Schrödinger evolution. In the variational approach, the conservation follows from the (formal) invariance of (84) $\phi(x, s) \mapsto \phi(x, s + s_0)$. However, the leakage to the sink on $\bar{\gamma}$, between γ_{s_1} and γ_{s_2} , is given by

$$8\pi^2 \bar{h}^3 \int_{s_1}^{s_2} ds \text{Re } \partial_s \phi^{\text{r}*}(\gamma_s, s) f^{\text{R}}(s), \quad (96)$$

is not guaranteed to vanish. Note that this leakage (whether positive or negative) is a ‘highly quantum’ phenomenon — proportional to \bar{h}^2 (the term $\partial_s \phi^{\text{r}}$ generally diverges as \bar{h}^{-1}).

Similarly, associated with the formal invariance of (84) under

$$A(x) \mapsto \lambda^{-1} A(\lambda^{-1} x), \quad \phi(x, s) \mapsto \lambda^{-2} \phi(\lambda^{-1} x, \lambda^{-2} s),$$

is a locally conserved dilatation current, the counterpart of the classical current (7),

$$\xi^\mu = p^{\mu\nu} x_\nu - \sum_k 2 \int_{-\infty}^{\infty} ds s^k B, \quad \text{with } B \text{ defined in (95)}. \quad (97)$$

The leakage to the sinks on $\bar{\gamma}$ is due to the second term, involving the mass-squared of the particles. A leakage of mass, therefore, also modifies the scale-charge of a solution.

B Spin- $\frac{1}{2}$ ECD

In a spin- $\frac{1}{2}$ version of ECD, the following modifications are made. The wave-function ϕ is a bispinor (\mathbb{C}^4 -valued), transforming in a Lorentz transformation according to

$$\rho(e^\omega)\phi \equiv e^{-i/4\sigma_{\mu\nu}\omega^{\mu\nu}}\phi, \quad \text{for } e^\omega \in SO(3,1), \quad (98)$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, with γ_μ Dirac matrices (not to be confused with γ the trajectory).

The propagator is now a complex, 4×4 matrix, transforming under the adjoint representation, satisfying

$$i\hbar\partial_s G(x, x', s) = \left[\mathcal{H} + \frac{g}{2}\sigma_{\mu\nu}F^{\mu\nu}(x) \right] G(x, x', s), \quad (99)$$

with the initial condition (16) at $s \rightarrow 0$ reading $\delta^{(4)}(x - x')\delta_{\alpha\beta}$, where $\delta_{\alpha\beta}$ is the identity operator in spinor-space, and g is some dimensionless ‘gyromagnetic’ constant of the theory.

The transition to spin- $\frac{1}{2}$ ECD is rendered easy by the observation that all expressions in scalar ECD are sums of bilinears of the form a^*b , which can be seen as a Lorentz invariant scalar product in \mathbb{C}^1 . Defining an inner product in spinor space (instead of \mathbb{C}^1)

$$(a, b) \equiv a^\dagger \gamma^0 b, \quad (100)$$

with γ^0 the Dirac matrix $\text{diag}(1, 1, -1, -1)$ (again, not to be confused with γ the trajectory) and substituting $a^*b \mapsto (a, b)$ in all bilinears, all the results of scalar ECD are retained. The Lorentz invariance of (100) follows from the Hermiticity of $\sigma^{\mu\nu}$ with respect to that inner product, viz. $(\sigma^{\mu\nu})^\dagger = \gamma^0 \sigma^{\mu\nu} \gamma^0$, and from $(\gamma^0)^2 = 1$.

Let us illustrate this procedure for important cases. By a direct calculation of the short- s propagator of (99), as in section 2.2.1, the spin can be shown to affect the $O(s)$ terms in the expansion of Φ , leading to an equally simple ϕ^s , the counterpart of (22), from which the regular part of all ECD currents can be obtained. The action, (84), from which all conservation laws can be derived, gets an extra spin term

$$\mathcal{A}_s[\varphi] = \int_{-\infty}^{\infty} ds \int_{\mathbb{M}} d^4x \frac{i\hbar}{2} [(\varphi, \partial_s \varphi) - (\partial_s \varphi, \varphi)] - \frac{1}{2} (D^\lambda \varphi, D_\lambda \varphi) + \frac{g}{2} (\varphi, F_{\lambda\rho} \sigma^{\lambda\rho} \varphi), \quad (101)$$

while the counterpart of the electric current, (11), derived from ϕ , is now a sum of an ‘orbital current’ and a ‘spin current’

$$j^\mu(x) \equiv j^{\text{orb}\mu} + j^{\text{spn}\mu} = \int ds \, q \text{Im}(\phi, D^\mu \phi) - g \partial_\nu (\phi, \sigma^{\nu\mu} \phi), \quad \text{for } x \notin \bar{\gamma}. \quad (102)$$

Expanding (102) in powers of ϵ , the coefficient of ϵ^0 is the regular current, j^r , associated with a particle. Each of the terms composing j^r is individually conserved and gauge invariant. The conservation of the orbital current follows from the $U(1)$ invariance of (101), while conservation of the spin current follows directly from the antisymmetry of σ . This current

has an interesting property that its monopole vanishes identically. Calculating in an arbitrary frame, using the antisymmetry of σ , and assuming $j^{\text{spn } i}(x) \rightarrow 0$ for $|\mathbf{x}| \rightarrow \infty$

$$\int d^3\mathbf{x} j^{\text{spn } 0} = \int d^3\mathbf{x} \int ds \partial_0(\phi, \sigma^{00}\phi) - \partial_i(\phi, \sigma^{i0}\phi) = 0 - 0 = 0. \quad (103)$$

The counterpart of (90) becomes (omitting the r identifier, as regularization is implied henceforth)

$$\partial_\nu ({}^k m^{\text{orb } \nu\mu} + g^{\nu\mu} {}^k \eta) = F^\mu{}_\nu {}^k j^{\text{orb } \nu} + \frac{g}{2} \int ds ({}^k \phi, \sigma^{\lambda\rho} {}^k \phi) \partial^\mu F_{\lambda\rho}, \quad \text{for } x \notin \bar{\gamma}, \quad (104)$$

with m^{orb} the same as (88) with $a^*b \mapsto (a, b)$ in all bilinears, and

$$l(x) = \frac{g}{2} \int ds (\phi, F_{\lambda\rho} \sigma^{\lambda\rho} \phi).$$

Note the ‘spin force’ density, vanishing for a constant F , which adds up to the Lorentz force density.

Similarly, adding $\int_{\mathbb{M}} d^4x l(x)$ to the functional in (91), equation (92) becomes

$$\partial_\nu \Theta^{\nu\mu} + \sum_k F^\mu{}_\nu {}^k j^{\text{orb } \nu} + \frac{g}{2} \int ds ({}^k \phi, \sigma^{\lambda\rho} {}^k \phi) \partial^\mu F_{\lambda\rho} + \partial_\nu g \int ds ({}^k \phi, \sigma^\nu{}_\lambda F^{\lambda\mu} {}^k \phi) = 0. \quad (105)$$

Summing (104) over k , and adding to (105), we get the locally conserved e-m tensor

$$\Theta^{\nu\mu} + \sum_k {}^k m^{\text{orb } \nu\mu} + g^{\nu\mu} {}^k \eta + g \int ds ({}^k \phi, \sigma^\nu{}_\lambda F^{\lambda\mu} {}^k \phi), \quad x \notin \cup_k {}^k \bar{\gamma}, \quad (106)$$

from which the time-independence of the associated charges follows as in the scalar case, as the extra terms involving spin, do not contain derivatives of ϕ .

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